The Geometric Supposer:

Quadrilaterals

Apple

SUNBURST
COMMUNICATIONS

CENTER FOR LEARNING TECHNOLOGY
EDUCATION DEVELOPMENT CENTER, INC.

SOFTWARE DESIGN: Judah L. Schwartz and Michal Yerushalmy

MANUAL DEVELOPMENT: Judah L. Schwartz, Michal Yerushalmy, and Myles Gordon

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This is THE GEOMETRIC SUPPOSER: Quadrilaterals, second in THE GEOMETRIC SUPPOSER software series created by Education Development Center, Inc. and published by SUNBURST COMMUNICATIONS, Inc. The first program in the series is THE GEOMETRIC SUPPOSER: Triangles.

About the Designers:

JUDAH L. SCHWARTZ is Professor of Engineering Science and Education at Massachusetts Institute of Technology, Senior Developer for the Center for Learning Technology at Education Development Center (EDC), and Visiting Professor at the Harvard Graduate School of Education. At Harvard, he is also co-director of the NIE-funded Educational Technology Center, a five year, $7.7 million effort to find new ways of using technology in science, mathematics, and computer science education. He developed SemCalc and is primary developer of the mathematics software series of the Center for Learning Technology, published through Sunburst Communications, Inc. In the past few years, Dr. Schwartz has become widely known as a writer, speaker, and for his thoughtful approach to computers and learning. His research interests include cognitive development in the learning of mathematics and problem-solving, uses of computers to augment human intuition, and the process and substance of undergraduate and continuing education. Previously, as co-director of the Cognitive Research Group at EDC, he conducted several research and demonstration projects designed to answer questions about how people learn to solve problems, and to investigate the use of microcomputers and printed materials to teach an approach to problem-solving. Dr. Schwartz holds an M.S. in Physics from Columbia University and a Ph.D. in Physics from New York University.

MICHAL YERUSHALMY is a Research Associate of the Center for Learning Technology at EDC. At EDC, she has worked closely with Dr. Schwartz on the mathematics software series. Her primary research interests are mathematics learning and teaching. She has conducted studies on new approaches to teaching mathematics and implementing specially designed software. Ms. Yerushalmy has over ten years of experience as a high school mathematics and computer science teacher and has served as a high school mathematics department chairman. In addition, she has taught procedural thinking and problem-solving to children in grades four through seven, using a special curriculum that introduces mathematical and logical concepts through graphics and games programming. Ms. Yerushalmy has B.S. degrees in Mathematics and Education and an M.S. degree in Education (the Department of Teaching Technology and Sciences) from the Technion in Haifa, Israel.

The CENTER FOR LEARNING TECHNOLOGY was established in 1982 by Education Development Center, which for over twenty-five years has been a pioneer in the use of new technologies as tools for teachers and learners. In addition to software development, the Center's activities include research, policy analysis, and videotape and videodisc production. The Center's mathematics software series, published by Sunburst Communications, Inc., addresses particularly troublesome areas of the mathematics curriculum at the elementary, middle, and high school levels. In addition to The Geometric Supposer series, programs include

Number Quest, which introduces search strategies with whole numbers, fractions, ordered number pairs, and ordered number triples; Word Quest, which explores the use of a mathematical problem-solving strategy in a non-mathematical context, building vocabulary and dictionary skills; Power Drill, which develops estimation skills in addition, subtraction, multiplication, and division; Get to the Point, which focuses on order of magnitude (i.e., where does the decimal point go?) in computation with decimals; and SemCalc, a tool for solving word problems.

The Center for Learning Technology is also developing software for writing instruction, programs in early reading instruction that use speech synthesis and recognition technologies, and a college level program in behavioral psychology that employs interactive videodisc. The Center for Learning Technology is a member of the Harvard-based, NIE-funded Educational Technology Center consortium.

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INTRODUCTION

There is something odd about the way we teach mathematics in our schools. We make little or no provision for students to play an active and generative role in learning mathematics and we teach mathematics as if we expected that students will never have occasion to invent new mathematics.

We don't teach language that way. If we did, we would never require students to write an original piece of prose or poetry. We would simply require them to recognize, appreciate, and memorize the great pieces of language of the past, literary equivalents of the Pythagorean Theorem and the Law of Cosines.

Most mathematics instruction is a kind of satire on the nature of mathematical thinking and the process of creating new mathematics. When a teacher assigns a theorem to prove, students assume that the theorem is true and that a proof can be found. The central activity of creating new mathematics--the making and the testing of conjectures--is absent from the classroom.

Making conjectures in geometry requires exploring the relationships that do or do not hold among geometrical "objects." And this exploration is made much easier if one can construct and manipulate these "objects." In the normal course of events, however, it is not feasible to ask students (or anyone else) to do a great deal of construction. Accurate constructions are difficult to make, and because of that difficulty, it is not reasonable to expect people to repeat constructions over and over again in order to generate a repertoire of cases on which to base a conjecture.

THE GEOMETRIC SUPPOSER is a series of programs designed to overcome these obstacles and thereby help the student become a potent and nimble conjecture-maker. Each program focuses on a broad section of the geometry curriculum. This program, the second in the series, is concerned primarily with the geometry of quadrilaterals.

THE GEOMETRIC SUPPOSER allows students to make any construction they wish on any quadrilateral. The program records that construction as a procedure that can then be executed with any other quadrilateral. As a result, the user can explore whether the consequences of a given construction on a given quadrilateral are dependent on some particular property of that quadrilateral, or if the result can be generalized.

Needless to say, neither possibility nor plausibility constitutes proof. Proof remains critical to both the creating and the learning of mathematics. But with the aid of THE GEOMETRIC SUPPOSER as intellectual amplifier, conjecture can assume its proper role as a key activity in the learning and teaching of geometry.

The fact that the SUPPOSER makes it possible to make constructions easily and quickly does not mean that the student should never be asked to use a straightedge and compass to bisect angles or to erect perpendiculars. Making constructions with tangible straightedges and compasses is essential. Direct experience with these tools makes the power of the SUPPOSER apparent.

What is THE GEOMETRIC SUPPOSER?

THE GEOMETRIC SUPPOSER is a microcomputer program that allows the user to carry out with ease constructions that are possible using straightedge and compass. These include the construction of quadrilaterals as well as the drawing of segments, parallels, perpendiculars, angle bisectors, and inscribed and circumscribed circles. In addition, the user can measure lengths, angles, areas, and distances as well as arithmetic combinations of these measures, such as the sum of two angles, the product of two lengths, the ratio of two areas, and the square of a segment length.

Such a program is useful in the learning of geometry in that it allows constructions to be made accurately and easily. The real power of THE GEOMETRIC SUPPOSER, however, lies in another feature--its ability to remember and to repeat constructions. Any construction on a quadrilateral that a user makes with the SUPPOSER may be repeated on a new quadrilateral of the user's construction, a previously used quadrilateral, a parallelogram (including rhombus, rectangle, or square), a trapezoid (including isosceles or right angle), or kite.

Using this feature, beginning geometry students can discover properties that appear to be true of particular constructions in particular quadrilaterals and properties that seem to be true when repeated on quadrilaterals of all shapes and sizes. Having established the plausibility of a conjecture, they can then devise a proof with conviction, growing out of some direct experience.

In so doing, students are coming to understand that mathematics is a lively and open-ended enterprise, and one that, with the right tools, is accessible.

How has THE GEOMETRIC SUPPOSER been used?

The SUPPOSER was designed originally for use in high school geometry classes. At the time of this writing, however, several middle school classes, as well as a dozen or so high school classes, both public and private, are using the program. In addition, at least one group of students in a vocational-technical high school is using THE GEOMETRIC SUPPOSER.

At the middle school level, students working with the SUPPOSER are exploring geometric ideas that are not part of the standard curriculum. The high school geometry classes are using the SUPPOSER in new ways to approach the traditional content of geometry courses. Some constitute rather modest departures from usual instruction, while others are truly revolutionary in the manner in which they draw mathematics out of the students. In the vocational setting, students are working with THE GEOMETRIC SUPPOSER to augment their studies in drafting and design and to strengthen their spatial reasoning skills.

In some cases, the SUPPOSER is being used in computer laboratories. In other cases, schools are using the program in classrooms with only one microcomputer. Needless to say, the availability of hardware dictates in part how the SUPPOSER is used, but teachers have demonstrated that the SUPPOSER can be used productively regardless of hardware constraints.

(For a more detailed discussion of using THE GEOMETRIC SUPPOSER in the classroom, see Section III of this manual.)

How was THE GEOMETRIC SUPPOSER developed?

The first version of THE GEOMETRIC SUPPOSER was developed during the 1981-82 school year. The program was first tried with students in the tenth grade at Commonwealth School in Boston in October 1982. Those early trials were exciting, challenging and frustrating. Although we believed deeply that we were engaged in an important endeavor in mathematics education, we were impatient with our own inability to clarify issues and define things as crisply as we would have liked.

In the Spring of 1983, the math faculty of the Weston (Massachusetts) High School and the administration of the Weston system asked us if we would consider using Weston as a site for pursuing further development of the SUPPOSER. With the collaboration of some remarkable teachers, we implemented THE GEOMETRIC SUPPOSER on the Weston School's minicomputer.

The 1983-84 school year saw the use of the SUPPOSER by Richard Houde and two of his geometry classes at Weston. From his classes, we learned a good deal about both the content and the teaching of geometry. As a result of that experience, conceptual issues were clarified and the operation of the program was simplified. The final version of the SUPPOSER reflects what we have learned from students and teachers.

During the 1984-85 school year, about fifteen teachers from Boston area schools have been using the SUPPOSER in a variety of settings. As a pilot users group, they have been meeting on a monthly basis to share their experiences and exchange ideas about the use of the program in the classroom.

An Afterword

We believe that THE GEOMETRIC SUPPOSER offers the mathematics teacher a new way to approach the teaching of geometry. More broadly, we believe that students can be active participants in the learning of mathematics and even make their own mathematics, and that microcomputers can help them to do so, thereby changing the way mathematics is learned and taught at all levels.

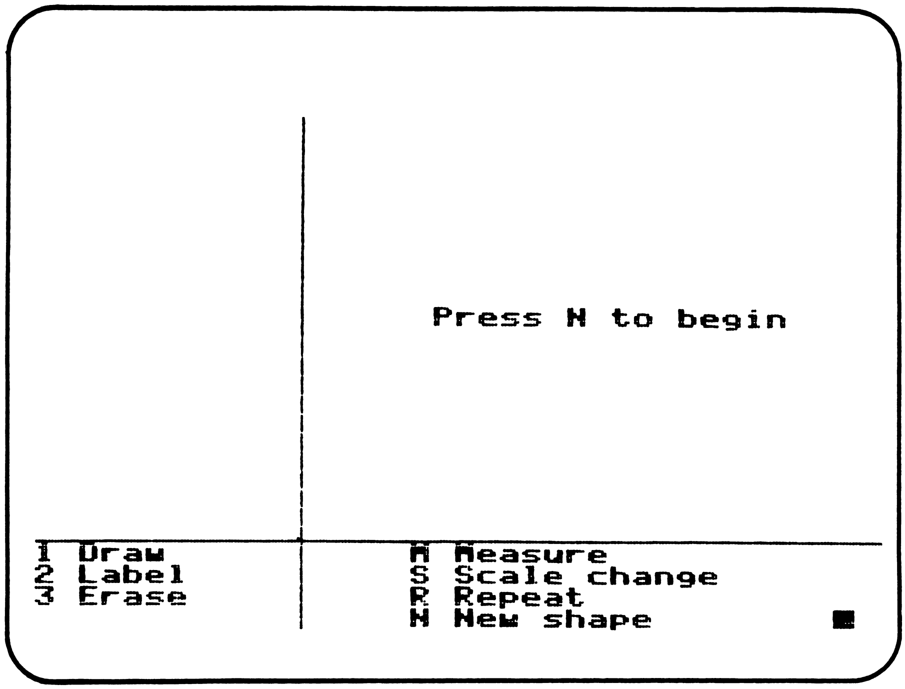
After more than three years of living with an intellectual undertaking that one nurtures and develops, it is difficult to say it is finished; but it is. However, the beliefs that led us to undertake development of the SUPPOSER can be articulated in other domains of mathematics. We turn eagerly now to that task.

Judah L. Schwartz
Michal Yerushalmy

I.1 HOW DO I GET STARTED?

Put THE GEOMETRIC SUPPOSER: Quadrilaterals disk into the disk drive, close the door of the drive, and turn on the computer and the monitor. While the program is loading, the Sunburst logo and the title screen will be displayed.

After the program has been loaded into memory, the screen will look like this:



The screen of the SUPPOSER is divided into three sections: the data column on the left side, the construction pad on the right side, and a window for menus and prompts at the bottom, below the horizontal line.

The message at the bottom of the screen is:

1 Draw	M Measure
2 Label	S Scale change
3 Erase	R Repeat
	N New shape

This is the MAIN MENU of the program and the one to which you will be returning over and over again. As indicated on the screen, you must press N for New Shape in order to proceed. At this point, the SUPPOSER will not respond to any other key.

I.2 HOW DO I MAKE A QUADRILATERAL?

After you press N, the message at the bottom of the screen will change to:

1 Parallelogram	4 Quads/Circles
2 Trapezoid	5 Your Own
3 Kite	

<u>New Shape</u> PARALELLOGRAM

1 Parallelogram

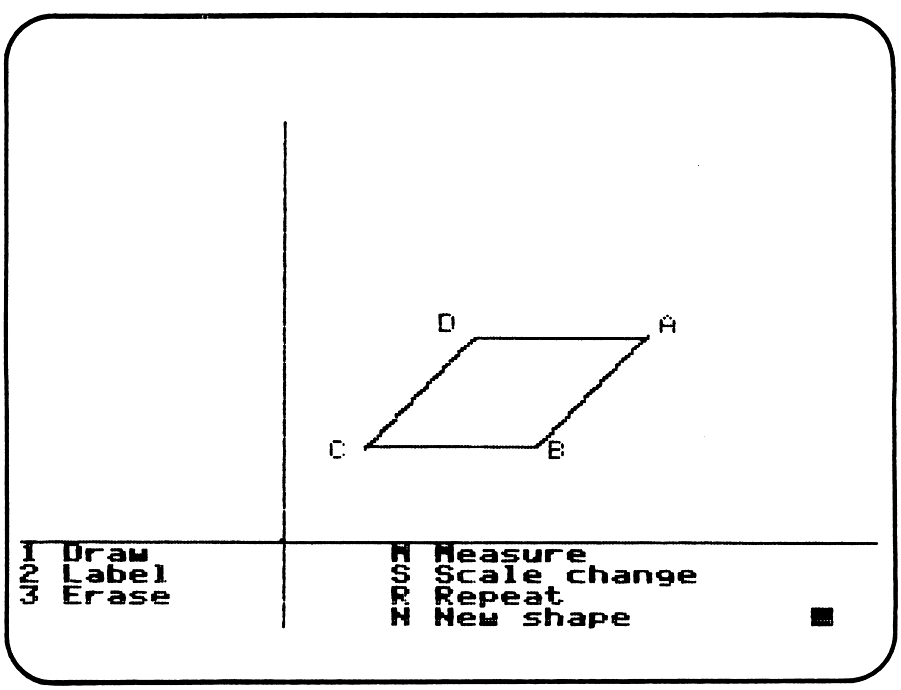
If you choose <u>1 Parallelogram</u>, the message at the bottom of the screen will read:

1 Random
2 Rectangle
3 Rhombus
4 Square

From this menu, you can select a parallelogram or subclass of parallelogram.

Suppose you wish to display a rhombus. After entering 1 for Parallelogram, enter the number 3 for Rhombus and the SUPPOSER will display a rhombus, label it ABCD, and return you to the MAIN MENU. You can now carry out constructions or measurements on your quadrilateral.

Here, for example, is a rhombus:



2 Trapezoid

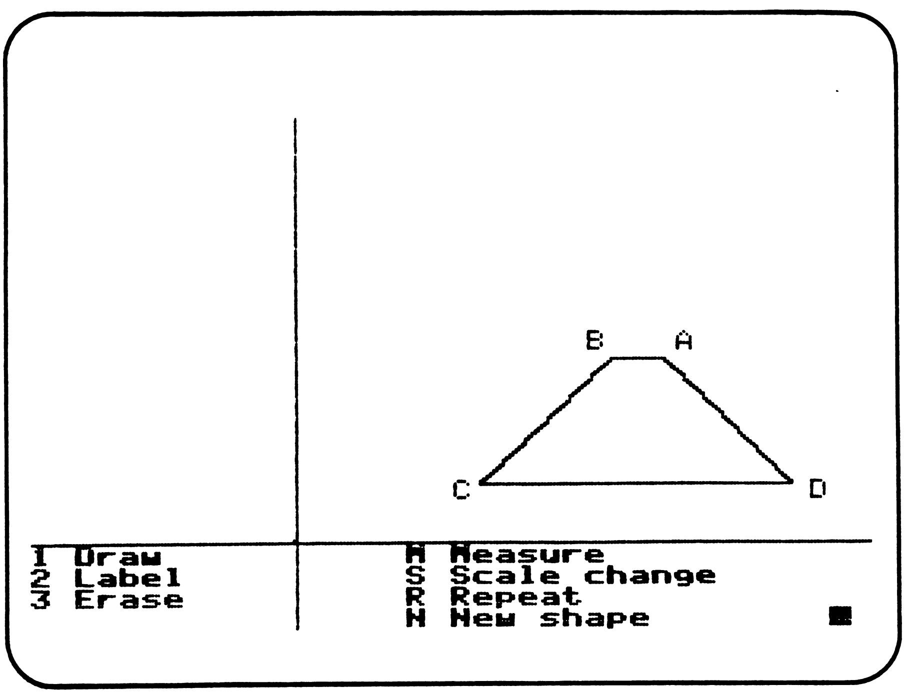
If you choose 2 Trapezoid, the message at the bottom of the screen will read:

1 Random
2 Isosceles
3 Right angle

From this menu, you can select a trapezoid or subclass of trapezoid.

Suppose you wish to display an isosceles trapezoid. After entering 2 for Trapezoid, enter the number 2 for Isosceles and the SUPPOSER will display an isosceles trapezoid, label it ABCD, and return you to the MAIN MENU. You can now carry out constructions or measurements on your quadrilateral.

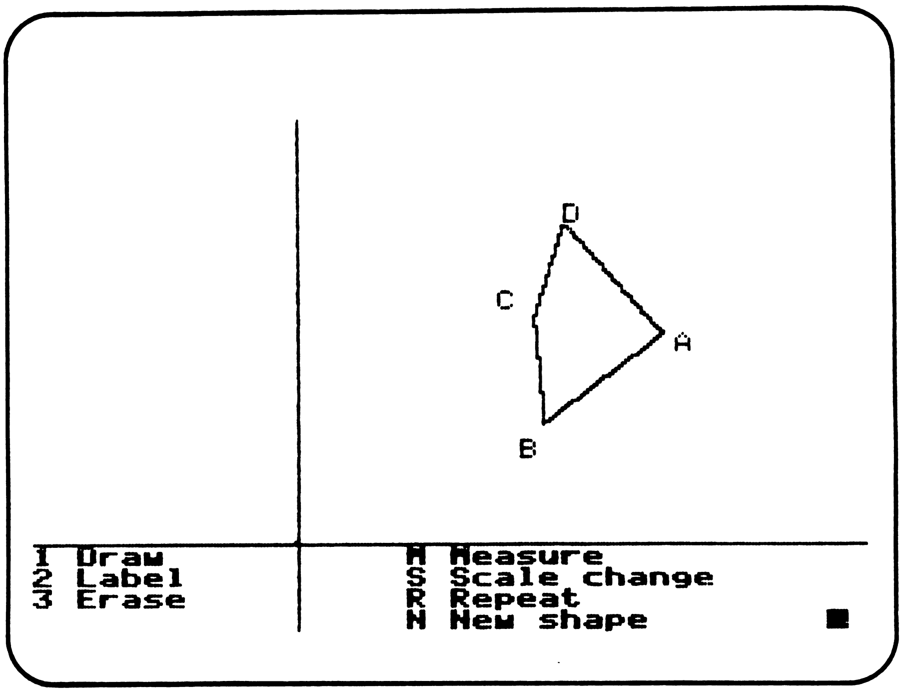
Here, for example, is an isosceles trapezoid:



3 Kite

If you choose 3 Kite, the SUPPOSER will generate a kite, label it ABCD, and return you to the MAIN MENU. (A kite is a quadrilateral that has reflection symmetry about one of its diagonals.)

Here is a kite:



4 Quads/Circles

If you choose 4 Quads/Circles, the message at the bottom of the screen will read:

Quadrilateral can:
1 contain an Inscribed circle
2 be Circumscribed by a circle

Not every quadrilateral can contain an inscribed circle, i.e., a circle that is tangent to all four sides of the quadrilateral. Similarly, not every quadrilateral can be circumscribed by a circle, i.e., a circle that passes through all four vertices of the quadrilateral.

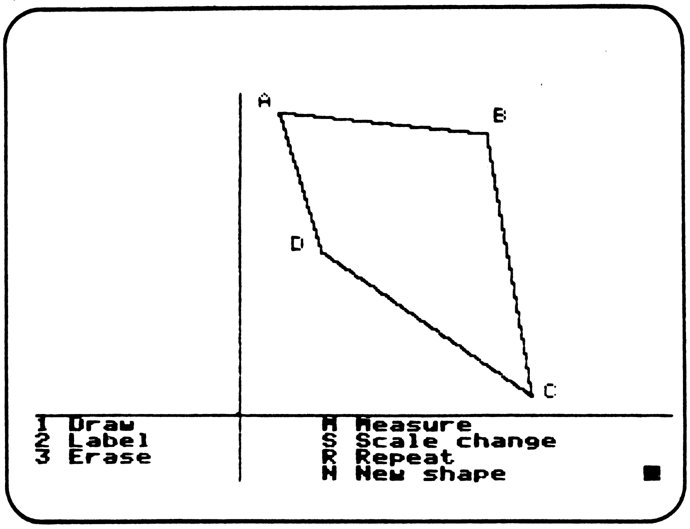
This choice allows you to generate a random quadrilateral that can have a circle inscribed within it or a quadrilateral that can be circumscribed by a circle.

Please note that while the options to create these classes of quadrilaterals are built into the SUPPOSER, you should consider asking your students at the appropriate time to construct such quadrilaterals without the use of these options.

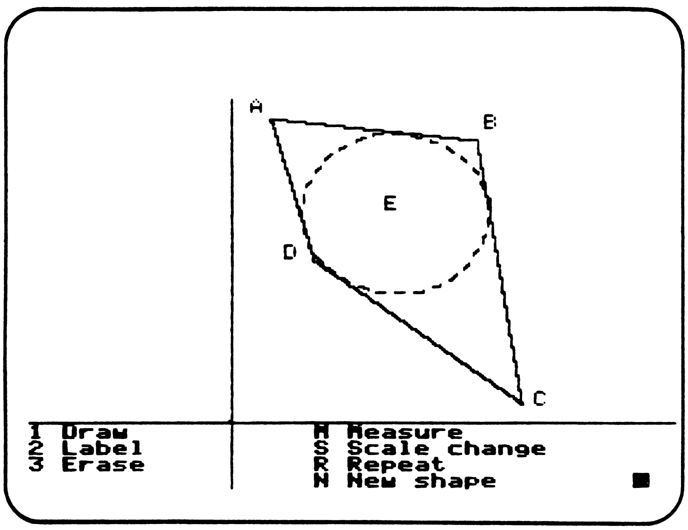
> Suppose you want to generate a quadrilateral that can contain an inscribed circle. After entering 4 for Quads/Circles, enter 1 (can contain an Inscribed circle), and the SUPPOSER will generate a quadrilateral that can contain an inscribed circle, label it ABCD, and return you to the MAIN MENU.

Here is:

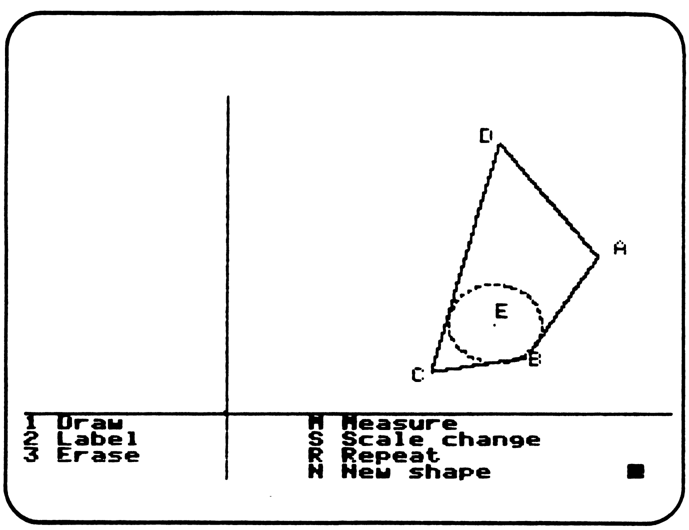
--a quadrilateral that can contain an inscribed circle



--the same quadrilateral with the circle constructed



--a quadrilateral that cannot contain an inscribed circle

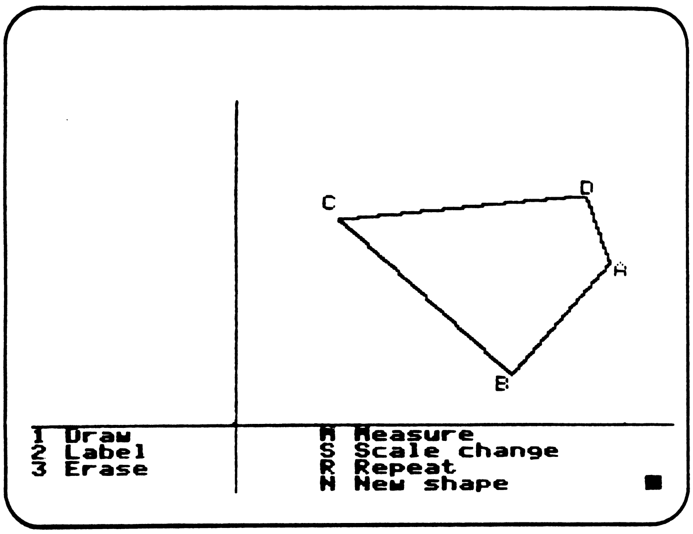


New Shape

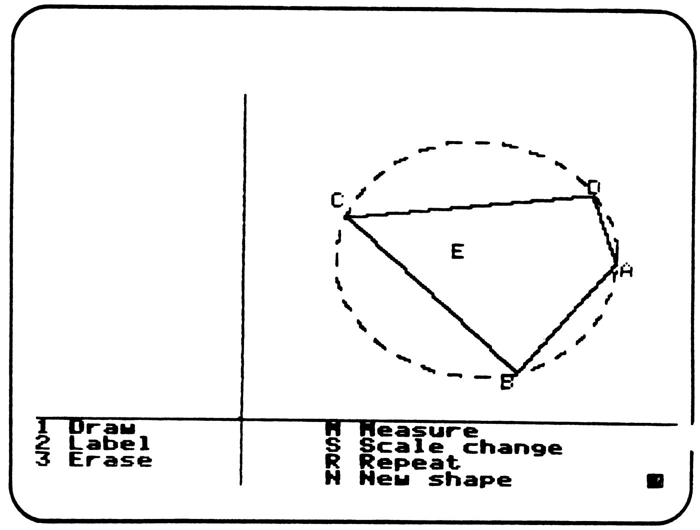
Suppose you want to generate a quadrilateral that can be circumscribed by a circle. After entering 4 for Quads/Circles, enter 2 (Circumscribed by a circle) and the SUPPOSER will generate a quadrilateral that can be circumscribed by a circle, label it ABCD, and return you to the MAIN MENU.

Here is:

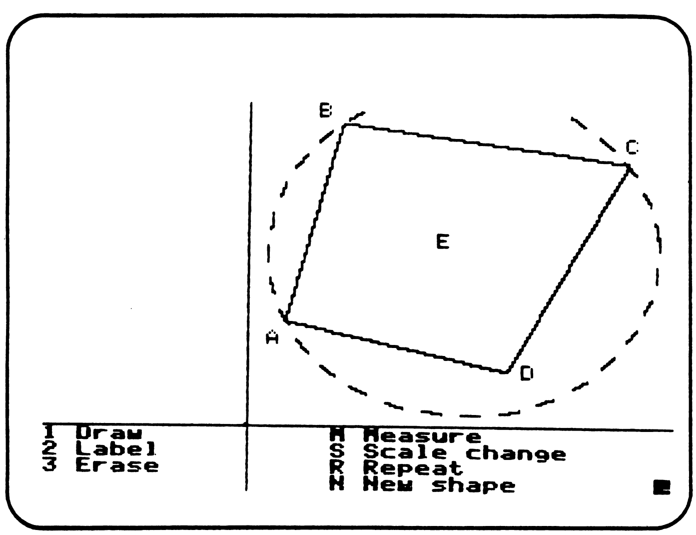
--a quadrilateral that can be circumscribed by a circle



--the same quadrilateral with the circle constructed



--a quadrilateral that cannot be circumscribed by a circle



5 Your Own

If you choose 5 Your Own, the message at the bottom of the screen will read:

Quadrilaterals constructed by:
1 Sides & Angles
2 Diagonals

These are two methods for defining a quadrilateral. 1 Sides & Angles allows you to specify the lengths of the sides and sizes of the angles necessary to construct a quadrilateral. 2 Diagonals allows you to specify the angles between the diagonals, the ratio of the lengths of the diagonals, and the point at which the diagonals intersect.

- Choose one by entering the corresponding number and the prompts for specifying your quadrilateral will appear at the bottom of the screen.

If you choose 1 Sides & Angles under Your Own, the SUPPOSER will ask you to specify quantities for two angles and two sides of the quadrilateral and then ask for a fifth quantity, the size of the third angle or the length of one of the two remaining sides.

A unit length will be drawn in the upper right-hand corner of the screen, labeled with the letter u. This length is the standard unit of length to be used for all constructions. (We suggest using sides with lengths less than about 9 units in order to be sure that your quadrilateral will fit on the screen.) If your quadrilateral turns out to be too small, you can use the rescale option (see Scale Change, pp. 84.)

In defining the lengths of sides and the sizes of angles, you may enter values to two decimal places.

Suppose you want to construct an isosceles trapezoid with one base that is 9 units long and two equal base angles of 70 degrees. After entering 1 for Sides & Angles, the message at the bottom of the screen will read

Angle DAB =
Side
Angle
Side

asking you to specify the size of angle DAB. Enter 70 and press RETURN. The SUPPOSER will construct an angle of 70 degrees at vertex A and ask for the length of side AB. Enter 9 for the length of AB and press RETURN. The

SUPPOSER will mark off a distance of 9 units along base AB and ask for the measure of angle ABC. Enter 70 for angle ABC and press RETURN. The SUPPOSER will construct an angle of 70 degrees at vertex B and ask for the length of BC. Since BC is not uniquely defined by the problem that we posed, say BC is 5 units long. Enter 5 and press RETURN. The SUPPOSER will mark off a distance of 5 units along BC and the message on the screen will now read

Angle DAB = 70	1 Angle BCD
Side AB = 9	2 Side AD
Angle ABC = 70	3 Side CD
Side BC = 5	

giving three choices for completing the specification of the quadrilateral.

Given the quantities specified so far, there are two ways to complete the construction of this isosceles trapezoid: specify that angle BCD measures 110 degrees (DAB + BCD = 180) or specify that AD is 5 units in length (BC=AD). In this case, let's assume we decide to specify the measure of angle BCD.

Press 1 to specify angle BCD and then enter 110 and press RETURN. The SUPPOSER will draw CD and the message will now read:

R - Reenter RETURN

R - Reenter allows you to start over and will return you to the menu for specifying your own quadrilateral by sides and angles.

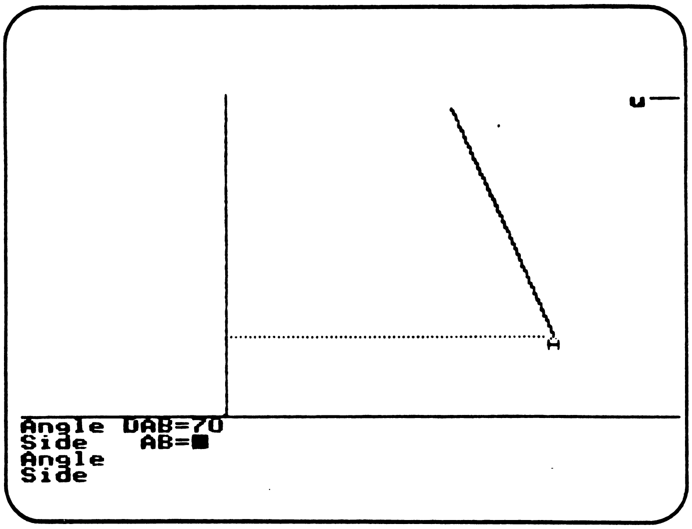
RETURN will erase the construction rays and arcs, will draw the quadrilateral you have just defined, and will return you to the MAIN MENU. You are now ready to carry out constructions or measurements on your quadrilateral.

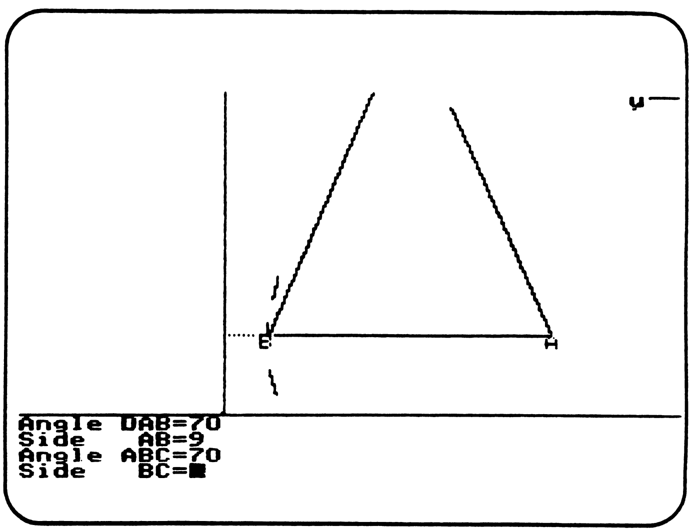
If a shape cannot be drawn from the quantities that you specify, the message on the screen will read

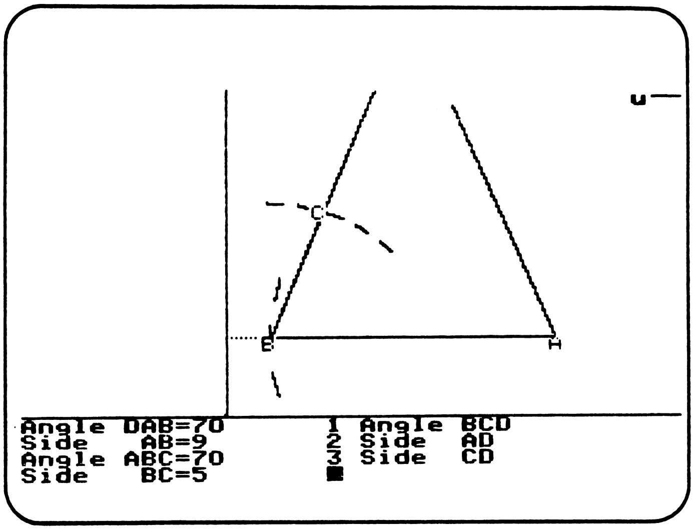
R - Reenter

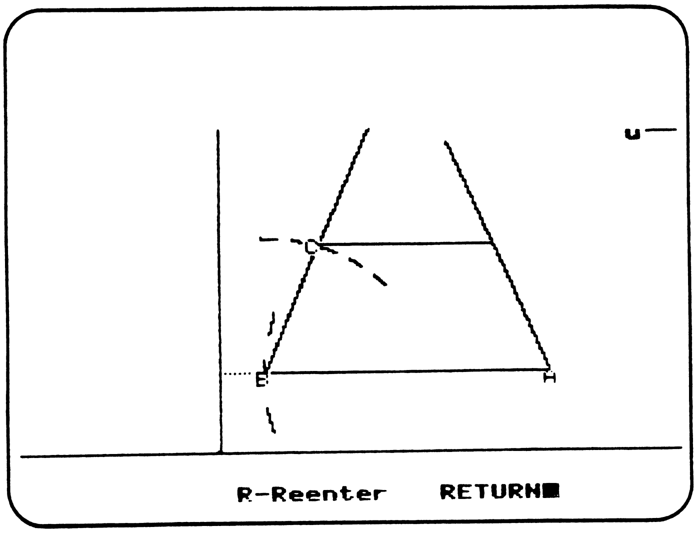
and after pressing R, the menu for specifying your shape by sides and angles will appear.

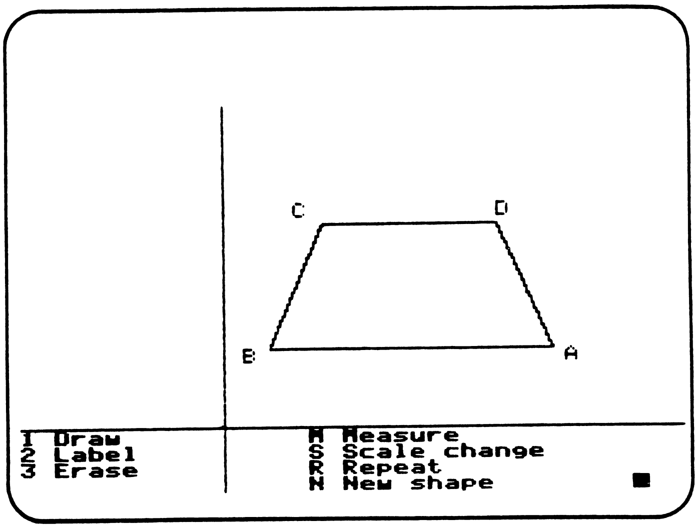
Here is the construction of an isosceles trapezoid having one base that is 9 units long and two equal base angles of 70 degrees:






C
B
A
Angle DAB=70
Side AB=9
Angle ABC=70
Side BC=5
1 Angle BCD
2 Side AD
3 Side CD


C
B
A
R-Reenter
RETURN


C
D
B
A
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New shape

If you choose 2 Diagonals under Your Own, the SUPPOSER will ask you to specify the angle between the diagonals in the quadrilateral, the ratio of the overall lengths of the two diagonals, and the ratio of each diagonal as it is cut by the other diagonal.

The idea that underlies this choice for constructing your own quadrilateral is as follows:

> Suppose you have two sticks and you place them on the table so that they cross one another. With the sticks in any position, you can join their endpoints with a string and form a quadrilateral. Moreover, if you fix the angles between the sticks, you can still create a large number of different quadrilaterals by sliding the sticks along each other.

Here is an example:

> Suppose you want to construct a quadrilateral with the following properties: The lengths of the diagonals are in the ratio 4:6 one of the angles between the diagonals is 90 degrees, and the ratios of the diagonals--defined by the point where they intersect--are 4:4 and 3:9 (a kite).
>
> After entering 2 for Diagonals, the message on the screen will read
>
> Angle between diagonals =
>
> asking you to define one of the angles between the diagonals. Enter 90 and press RETURN. The SUPPOSER will draw two provisional diagonals, labeled AC and BD, at an angle of 90 degrees to one another. The message will will now read
>
> Angle between diagonals = 90
Length AC:Length BD = :
>
> asking you to define the ratio of the overall lengths of diagonals AC and BD. Enter 4 and press RETURN; then enter 6 and press RETURN. The SUPPOSER will draw the two diagonals in the ratio that you have specified and the message at the bottom of the screen will read:
>
> <AEB =90
AC:BD = 4:6
Arrows to move intersection point

Note that we could draw segments AB, BC, CD, and DA now and we would have a quadrilateral with an angle of 90 degrees between its diagonals and the ratio of the lengths of its diagonals 4:6.

There is, however, another variable which can be considered in the construction of the quadrilateral, i.e., the ratio of the parts of each diagonal defined by where it intersects the other diagonal. We can slide each diagonal parallel to itself and still have a quadrilateral with diagonals in a ratio of 4:6 and with an angle between the diagonals of 90 degrees.

You can slide the diagonals over one another using the following keys:

	APPLE IIe or IIc	APPLE II+
LEFT	left arrow	left arrow
RIGHT	right arrow	right arrow
UP	up arrow	CTRL K
DOWN	down arrow	CTRL J

As you use the keys indicated above to move the point where the two diagonals intersect, the SUPPOSER will display the changing values of the ratios of each diagonal (AE:EC and BE:ED, where E is the point of intersection), indicating where along their lengths the two diagonals cut one another:

```
<AEB =90          AE:EC=4:4
AC:BD=4:6         DE:EB=11:13

                   D-Draw  R-Reenter
```

When the diagonals are in the ratios you want--in this case:

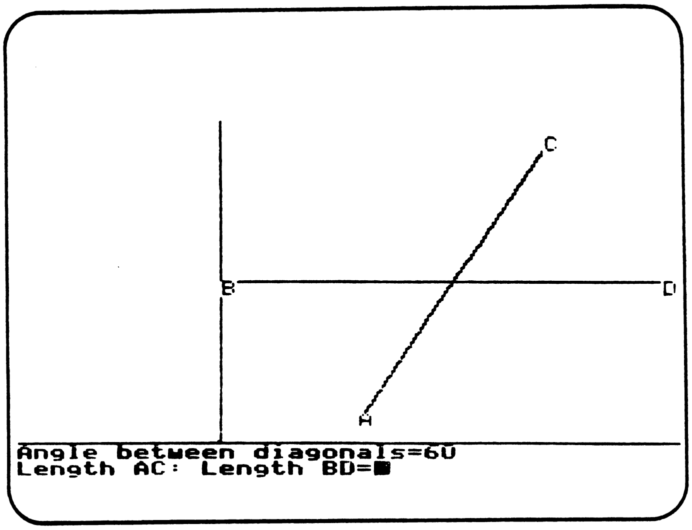
```
<AEB =90          AE:EC=4:4
AC:BD=4:6         DE:EB=3:9

                   D-Draw  R-Reenter
```

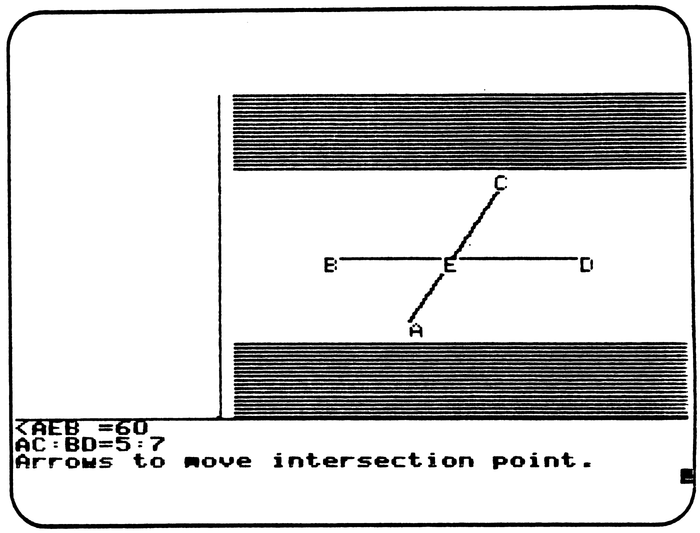
press D (for Draw) and the SUPPOSER will draw the quadrilateral that you have specified and return you to the Main Menu to carry out constructions or measurements on your quadrilateral.

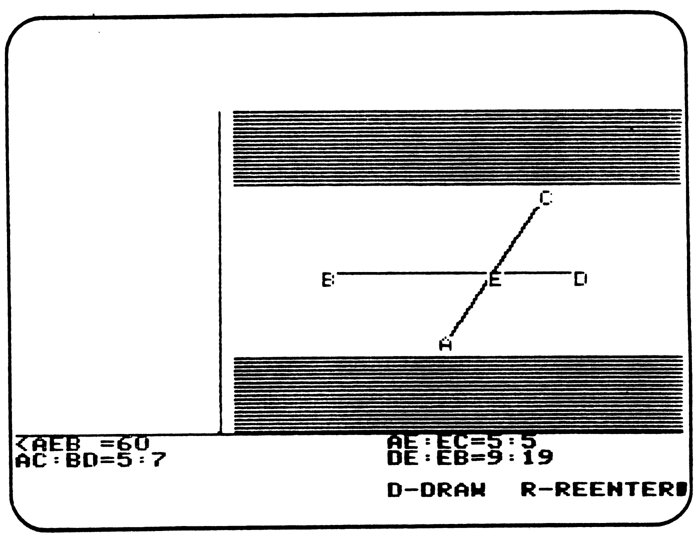
If you press R (for Reenter), the SUPPOSER will return you to the menu for specifying your own quadrilateral by diagonals and you can start over.

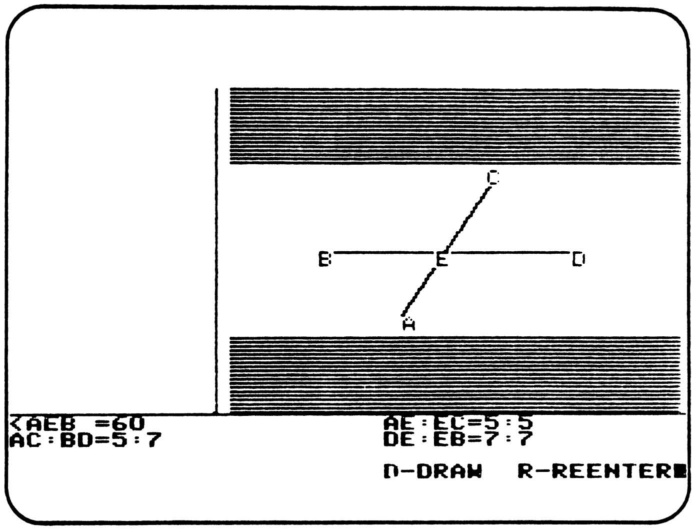
<u>New Shape</u> YOUR OWN

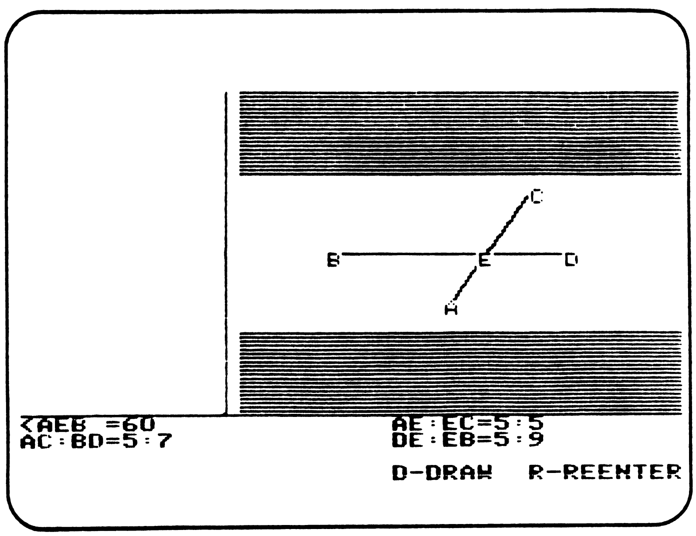
Here is the construction of a quadrilateral with the angles between the diagonals being 60 degrees, the overall lengths of the diagonals in the ratio 5:7, and the ratios of the diagonals where they intersect, 5:5 and 5:9:

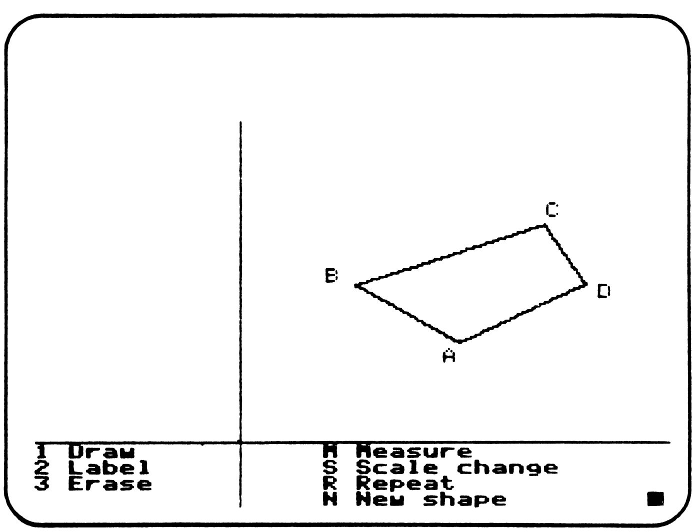






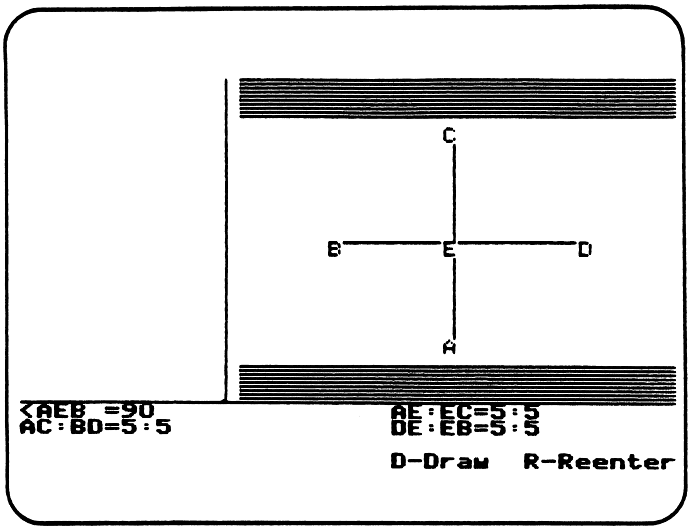

C
B
E
D
A
<AEB =60
AC:BD=5:7
AE:EC=5:5
DE:EB=7:7
D-DRAW R-REENTER


C
B
E
D
A
<AEB =60
AC:BD=5:7
AE:EC=5:5
DE:EB=5:9
D-DRAW R-REENTER

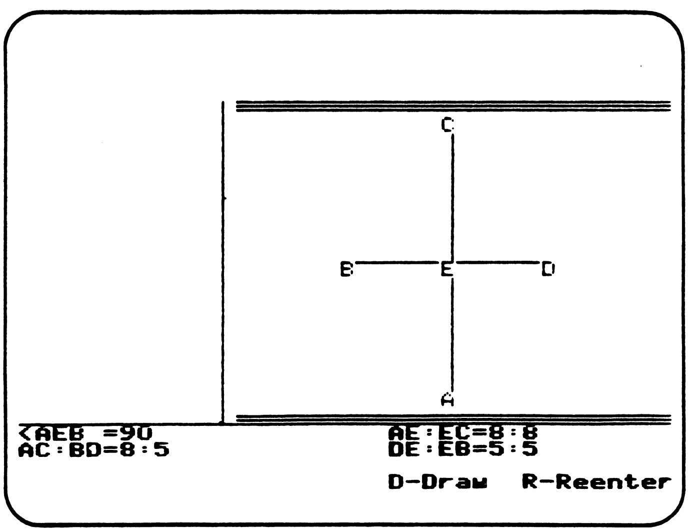

C
B
D
A
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New shape

The following descriptions and figures provide the specifications by diagonals for different kinds of quadrilaterals.

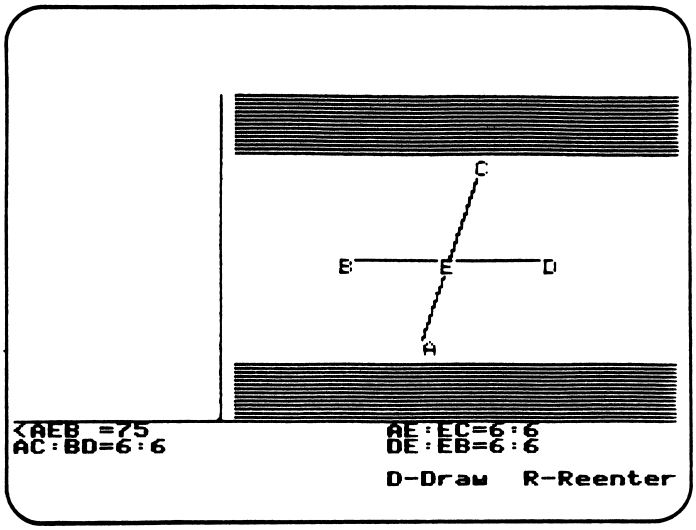
If the angle between the diagonals is a right angle (90 degrees), the two diagonals are of equal length, and both diagonals are bisected, the quadrilateral is a SQUARE.



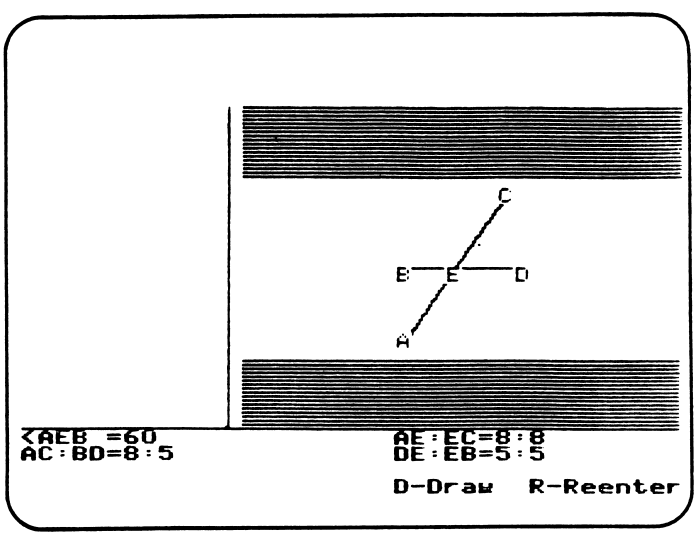
If the angle between the diagonals is a right angle (90 degrees), the diagonals are not equal, and both diagonals are bisected, the quadrilateral is a RHOMBUS.



If the angle between the diagonals is not a right angle (90 degrees), the diagonals are equal, and both diagonals are bisected, the quadrilateral is a RECTANGLE.



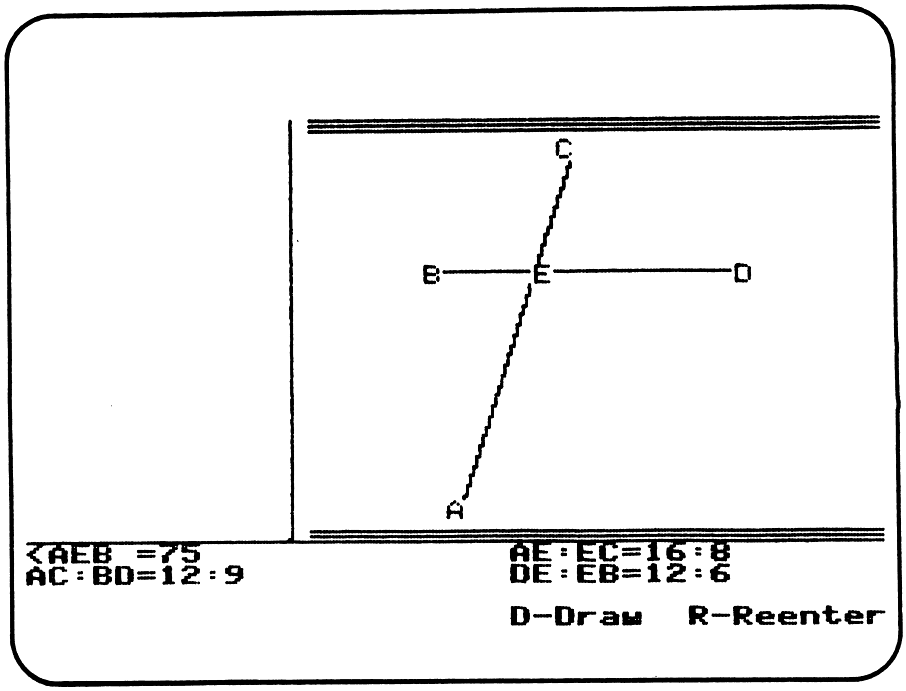
If the angle between the diagonals is not a right angle (90 degrees), the diagonals are not equal, and both diagonals are bisected, the quadrilateral is a PARALLELOGRAM.



New Shape

YOUR OWN

If diagonals intersect so that the ratios of the segments are equal, the quadrilateral is a TRAPEZOID.



I.3 THE GEOMETRIC SUPPOSER--AN OVERVIEW OF THE MAIN MENU

The MAIN MENU displays across the bottom of the screen as follows:

1 Draw	M Measure
2 Label	S Scale change
3 Erase	R Repeat
	N New shape

To choose one of these options, enter the corresponding number or letter and you will be presented with the appropriate submenu.

In this section, each of these options will be described briefly to give you an overview of THE GEOMETRIC SUPPOSER. In the next section of the manual, each of the options and the suboptions will be described in much greater detail.

<u>Pressing ESC returns you to the previous menu.</u>

1 Draw

If you choose option 1 in the MAIN MENU, Draw, the message at the bottom of the screen changes to:

1 Segment	4 Angle Bisect
2 Circle	5 Parallel
3 Extension	6 Perpendicular

These are the possible constructions that you may draw.

> To make any one of these constructions, enter the appropriate number and the submenu for that construction will appear on the screen.

The operation of each of these choices is explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

2 Label

If you choose option 2 in the MAIN MENU, Label, the message at the bottom of the screen changes to:

1 Intersection
2 Subdivide segment
3 Reflection
4 Random Point

This option allows you to label intersections, subdivide line segments, reflect portions of a figure in a line, and generate a random point.

> To carry out one of these operations, enter the appropriate number and the submenu will appear on the screen.

The operation of each of these choices will be explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

3 Erase

If you choose option 3 in the MAIN MENU, Erase, the message at the bottom of the screen changes to:

1 Erase segment:
2 Erase label(s):

This option allows you to clear a segment(s) from the screen and to unclutter a construction by erasing a label(s) on the screen. Although segments and labels will not appear on the screen once erased, they remain in the memory and can be used to carry out constructions and to take measurements.

The operation of each of these choices is explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

M Measure

If you choose option M on the MAIN MENU, Measure, the message at the bottom of the screen changes to:

1 Length	4 Angle
2 Perimeter	5 Distance Point-Line
3 Area	6 Distance Line-Line

This option allows you to measure lengths, perimeters, areas, and angles; and the distances between points and lines, and between lines and lines. You can also measure the sum, difference, product, or ratio of two lengths, perimeters, areas, angles, or distances. You can square the measure of a length, perimeter, area, angle, or distance as well. The values of the measured quantities are printed in the Data Column on the left-hand side of the screen.

To make one of these measurements, enter the appropriate number and the submenu will appear on the screen.

The operation of each of these choices is explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

S Scale Change

There are two sizes available for each image. If you choose option S in the MAIN MENU, Scale Change, the image on the screen is redrawn in the other of the two possible sizes.

This option has another function as well. If a construction cannot be fully displayed on the screen and the beep sounds, try using the Scale Change option. This option, in addition to changing the scale of the quadrilateral, checks the placement of the quadrilateral on the screen. In most cases, the program can relocate the quadrilateral so that the construction can be displayed.

> To carry out a scale change, enter the letter S and the SUPPOSER will rescale the quadrilateral.

Pressing ESC returns you to the previous menu.

R Repeat

If you choose this option in the MAIN MENU, the message across the bottom of the screen will change to:

Repeat construction:
1 on new shape
2 on previous shape

You may now repeat the construction you have just made on a new quadrilateral or on any one of your three most recent quadrilaterals.

The operation of each of these options is explained in detail in the next section of the manual.

Pressing ESC returns you to the previous menu.

N New Shape

If you choose this option in the MAIN MENU (by pressing N), then you will be returned to the menu of possible quadrilaterals described in Section I.2 (HOW DO I MAKE A QUADRILATERAL?).

Pressing ESC returns you to the previous menu.

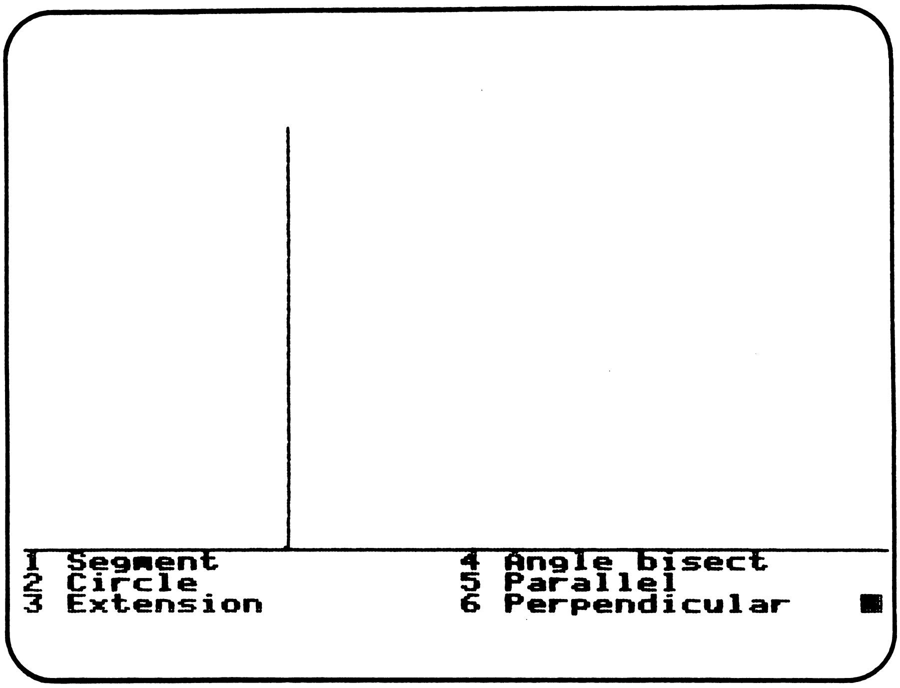
II. USING THE GEOMETRIC SUPPOSER--THE SUBMENUS

The heart of using THE GEOMETRIC SUPPOSER lies in its submenus. This section of the manual describes each choice in each submenu of the MAIN MENU.

At this point we suggest that you refer to the reference card included with the program; it provides a chart of the various submenus and essential information about the operation of the SUPPOSER.

DRAW

After you have created a quadrilateral, you can make constructions on that quadrilateral by using the Draw option. Selecting Draw will result in the following submenu screen:



Draw

1 Segment

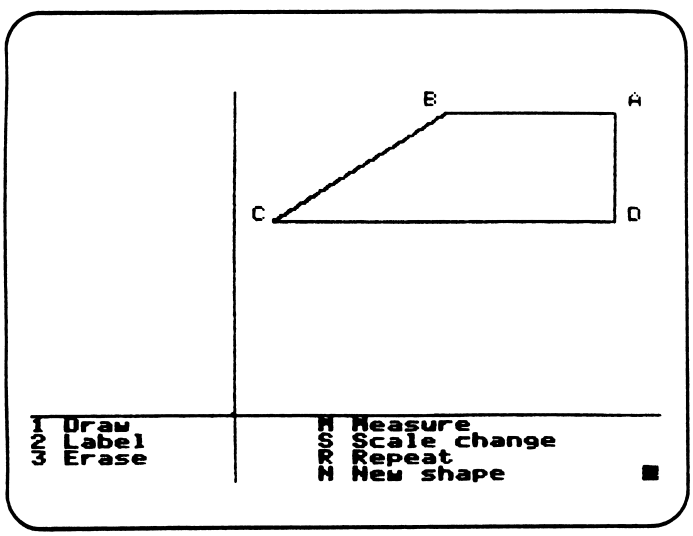
Choice 1 under Draw is Segment. If you choose this option, the SUPPOSER will ask what segment you want to draw. You may connect any two labeled points on the screen.

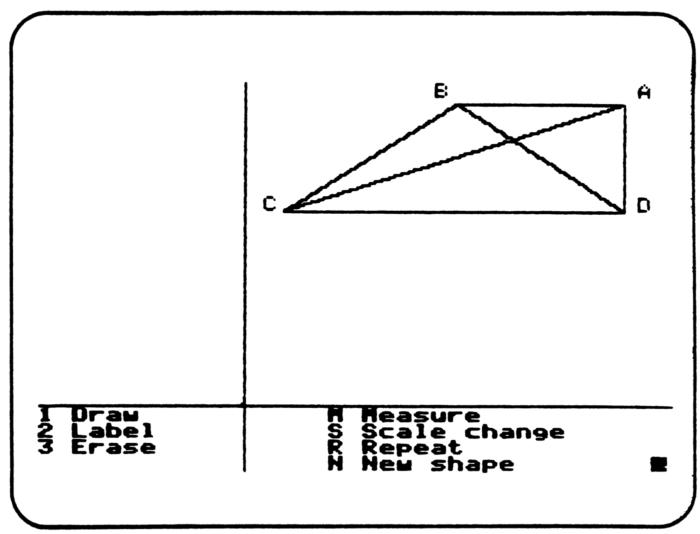
After entering the number 1, the message at the bottom of the screen will read

Segment name:

asking you to identify the names of the two points you want to connect with a segment. Enter the name of the segment and the SUPPOSER will draw the segment and return you to the MAIN MENU.

For example, here is a right angle trapezoid ABCD and the same quadrilateral with its diagonals, AC and BD, drawn using this option:





2 Circle

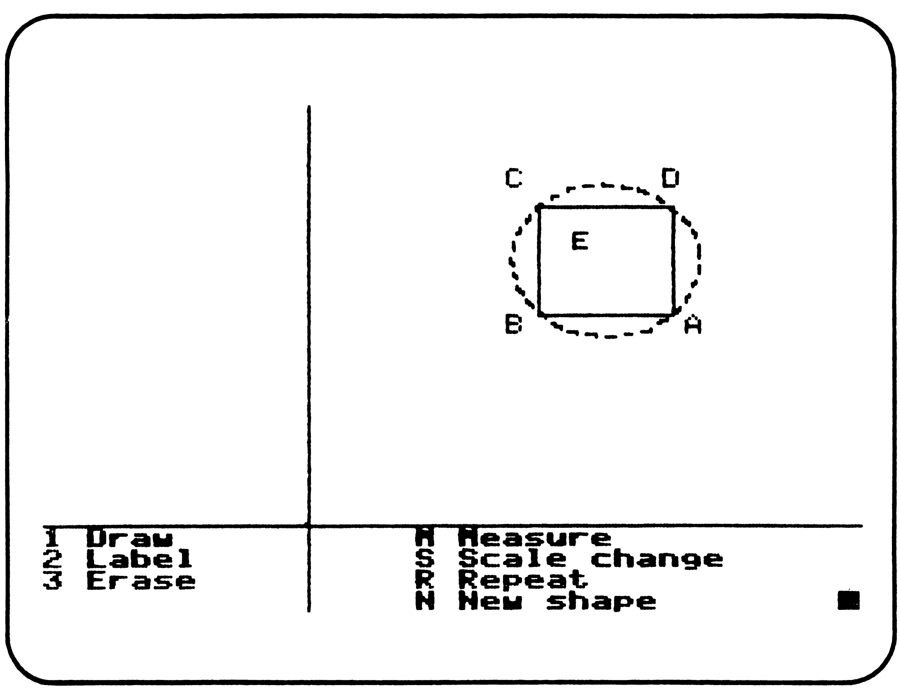
Choice 2 under Draw is Circle. If you choose this option, the SUPPOSER will ask whether you want to draw a circumscribed or inscribed circle, or to define your own circle by specifying the center of the circle and the length of the radius. By entering three letters, the circle will be drawn in relation to a triangle defined by three labelled points on the screen. By entering four letters, the circle will be drawn in relation to a quadrilateral defined by four labelled points on the screen. After entering the number 2, the message on the screen will read:

1 Circumscribes shape:
2 Inscribed in shape:
3 Other

A circle that circumscribes a polygon passes through all the vertices of the polygon.

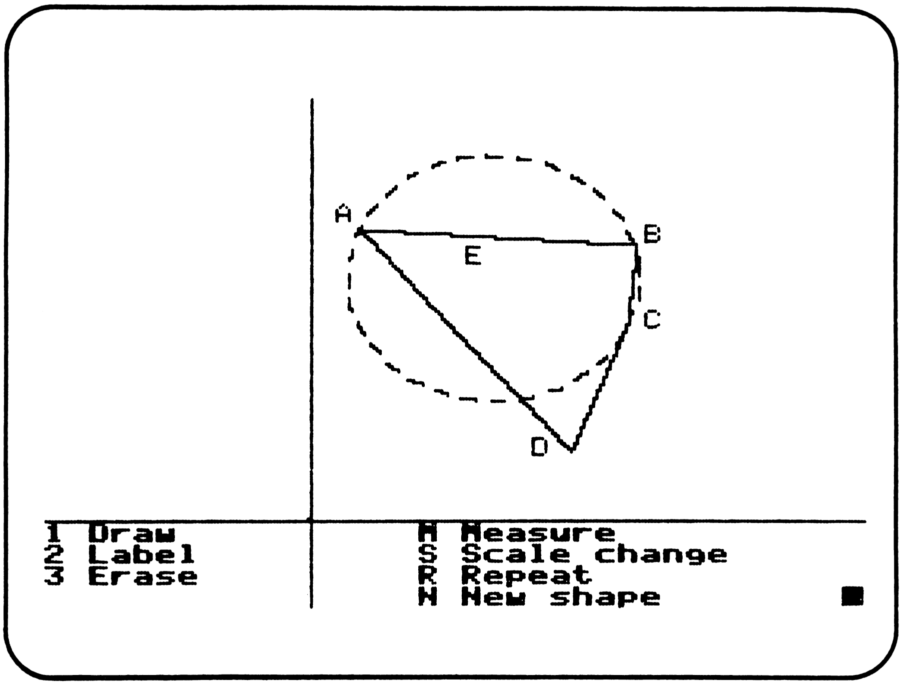
Suppose you want to circumscribe square ABCD. After entering 1 for Circumscribes, enter the name of the quadrilateral which you would like the circle to circumscribe (e.g., ABCD) and press RETURN. The SUPPOSER will draw the circle, label its center (E), and return you to the MAIN MENU.

Here is a circle that circumscribes square ABCD:



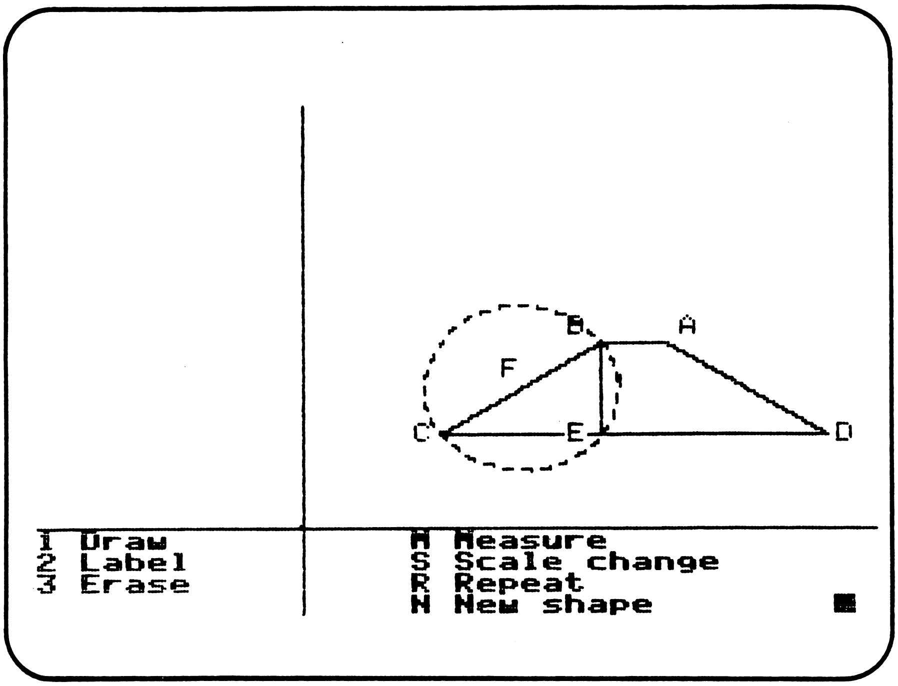
Not all quadrilaterals, however, can be circumscribed by a circle. If you choose such a quadrilateral, the SUPPOSER will circumscribe the triangle defined by the first three letters of the quadrilateral, label the center and return you to the MAIN MENU.

Here is what happens when you try to circumscribe a quadrilateral that cannot be circumscribed:



Suppose you want to circumscribe triangle BEC created by drawing a perpendicular through vertex B in isosceles trapezoid ABCD. After entering 1 for Circumscribes, enter the name of the triangle which you would like the circle to circumscribe (BEC) and press RETURN. The SUPPOSER will draw the circle, label its center (F), and return you to the MAIN MENU.

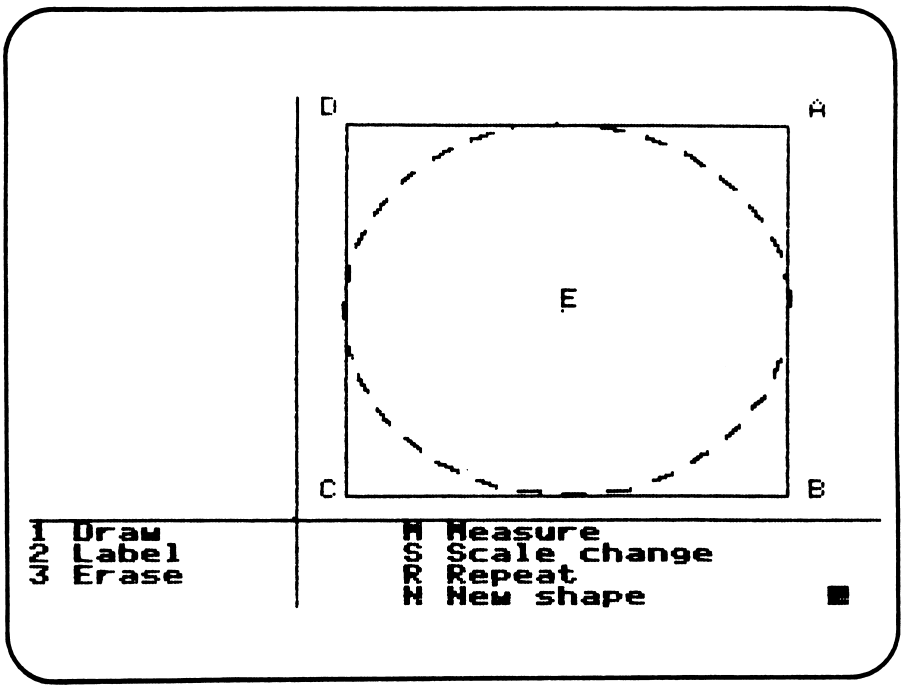
Here is a circle that circumscribes triangle BEC in trapezoid ABCD:



A circle that is inscribed in a polygon is tangent to all sides of the polygon.

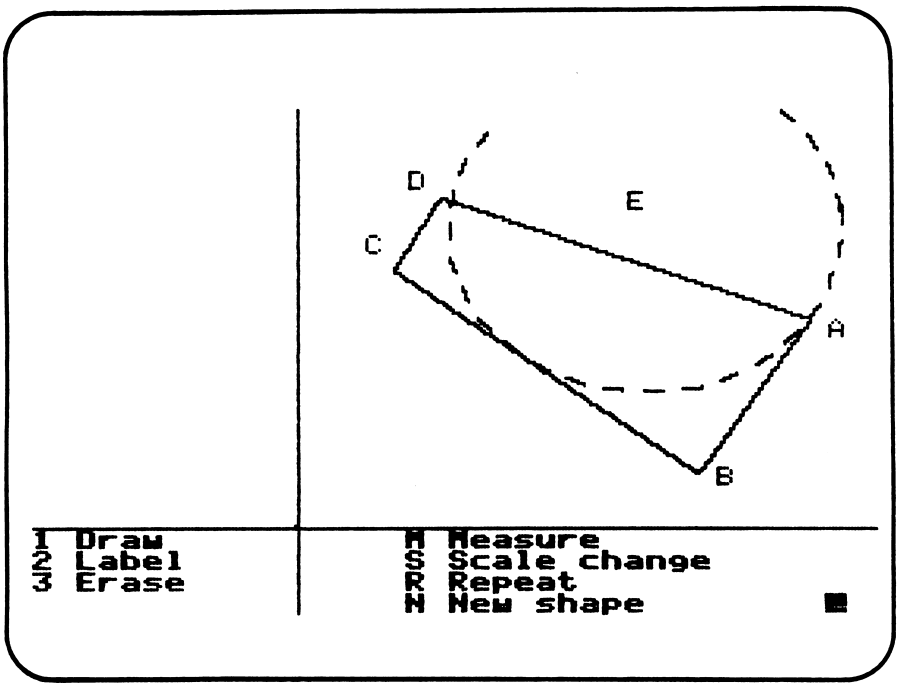
Suppose you want to inscribe a circle in square ABCD. After entering 2 for Inscribed, enter the name of the quadrilateral in which you would like the circle to be inscribed (e.g., ABCD) and press RETURN. The SUPPOSER will draw the circle, label its center (E), and return you to the MAIN MENU.

Here is a circle inscribed in square ABCD:



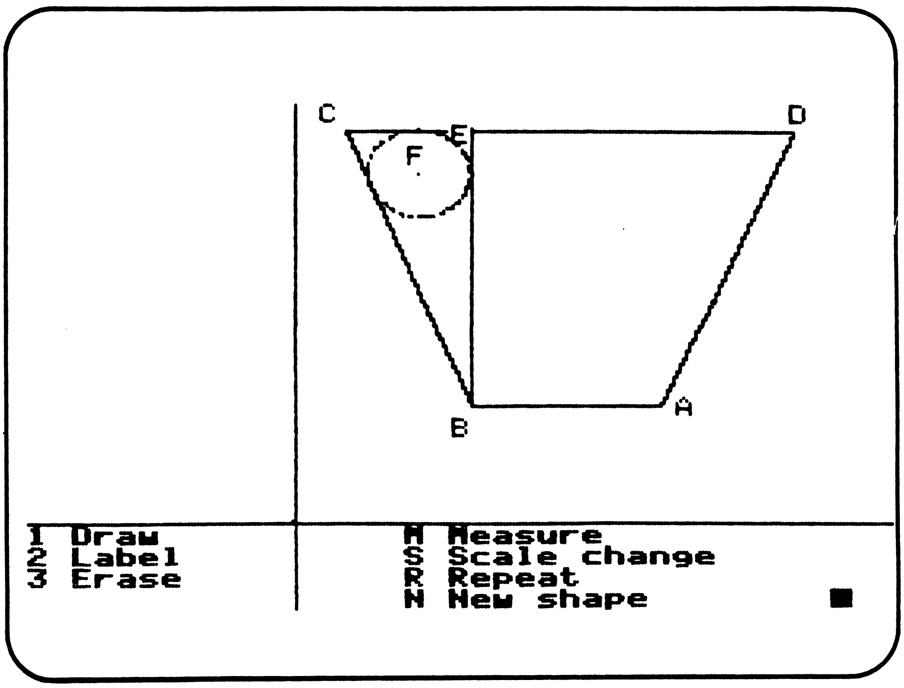
Not all quadrilaterals, however, can be inscribed by a circle. If you choose such a quadrilateral, the SUPPOSER will inscribe a circle tangent to as many line segments as possible, label the center, and return you to the MAIN MENU.

Here is what happens when you try to inscribe a circle in a quadrilateral when it can't be done:



Suppose you want to draw an inscribed circle in triangle BEC created by drawing a perpendicular through vertex B in isosceles trapezoid ABCD. After entering 2 for Inscribed, enter the name of the triangle in which you would like to inscribe the circle (BEC) and press RETURN. The SUPPOSER will draw the circle, label its center (F), and return you to the MAIN MENU.

Here is a circle inscribed in a triangle in an isosceles trapezoid:



Choice 3, Other, allows you to draw a circle with any labeled point on the screen as its center and with a radius of your selection.

> For example, suppose you wish to construct a circle on isosceles trapezoid ABCD with point A serving as its center, and with a radius equal to side AD.
>
> After entering the number 3, the following message will appear on the screen:
>
> Circle's center:
>
> Now enter the name of the point that will serve as the center of the circle (A).

Any labeled point on the screen may serve as the center of the circle. If the point that you wish to be the center of your circle has no label, you may place a point there and label it. See the options in the Label menu.

> After you identify the center of your circle (in this case, A), you must specify the length of the radius of your circle.

You can specify the radius in terms of 1) some constant multiple of the length of any segment depicted on the screen, or 2) some multiple of the unit length u which appears in the upper right-hand corner of the screen.

> The message on the screen will now read:

```
Circle's center: A
Radius =(segment or unit) * constant
       =                  *
```

> To construct a circle with a radius equal to the length of AD, enter AD for segment or unit. The message will now read:

```
Circle's center: A
Radius =(segment or unit) * constant
       = AD               *
```

> Now for constant, enter 1, press RETURN, and the SUPPOSER will draw the circle you have defined and return you to the MAIN MENU.

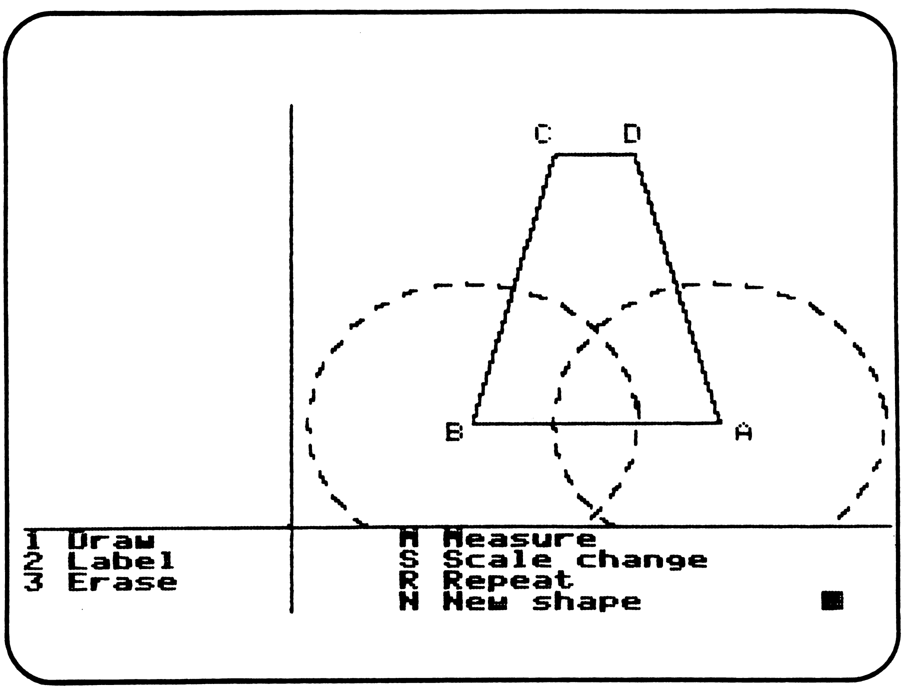
If instead you want to define the radius in terms of some multiple of the unit length, u, enter u for segment or unit and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw the circle.

To illustrate the distinction between these two ways of specifying a radius, consider the following example and figures. Quadrilateral ABCD is an isosceles trapezoid with side lengths AD = BC = 8. Suppose we draw a circle centered on B with a radius of AD * .5, and a circle of radius 4 centered on A.

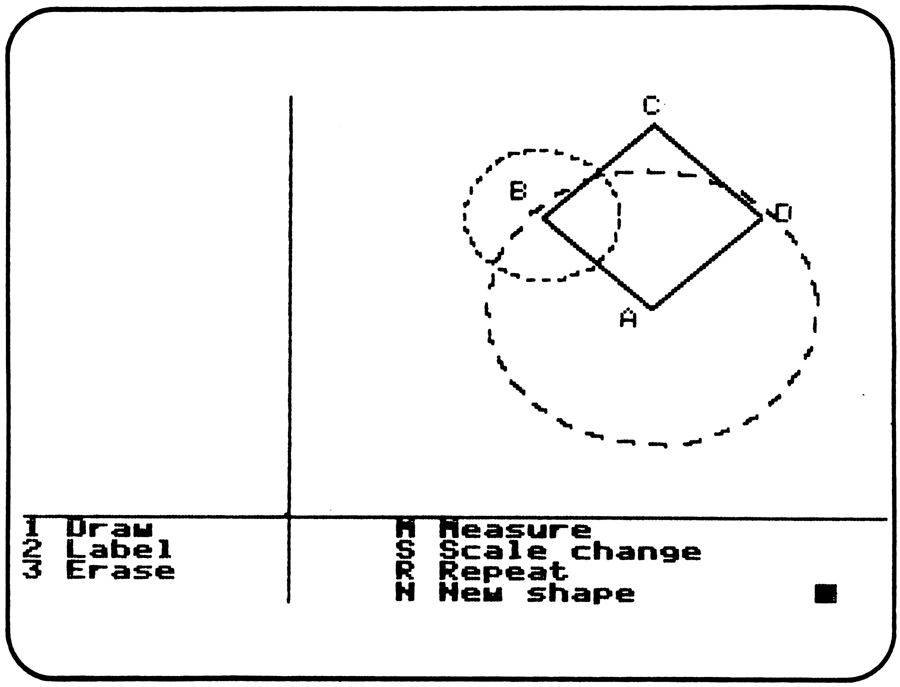
The radius of the circle centered on B is defined in terms of the length of the segment AD. The radius of the circle centered on A is defined in terms of a certain number of screen units. Even though the two radii have the same numerical value, when we repeat the construction on a different quadrilateral or if we rescale the drawing, the two circles will behave differently.

If the construction of the two circles is repeated on another quadrilateral, the radius of the circle centered on B remains one half of the length of the segment AD, while the radius of the circle centered on C is still 4 units.

Here are the circles constructed on isosceles trapezoid ABCD:



In this figure, the construction of the circles has been repeated on a square. Note the difference in the size of the circle centered on B (with its radius defined in terms of AD) when the construction is repeated.



3 Extension

Choice 3 under Draw is Extension. This option allows you to extend a line segment from either end. The SUPPOSER will ask you to specify which line segment you wish to extend and from which end you wish to extend it. You will then be asked to specify the length of the extension you wish to draw in terms of 1) a segment or the unit length, or 2) to intersect segments on the screen.

Suppose you want to extend side AD from point D, with the extension equal in length to side BC. After entering the number 3, the message on the screen will read:

Extend segment:

Enter the name of the segment you want to extend (BC) and the message will now read:

Extend segment: AD
From point:

Enter the name of the point from which you want the segment to extend (D) and the message will change to:

Length defined:
1 by (segment or unit) * constant
2 to intersect segment(s)

Enter the number 1 to define the length of the extension by a length equal to segment BC. Now enter BC for segment or unit length. The message will now read:

Length defined:
(segment or unit) * constant =
BC *

Now enter 1 for the constant and press RETURN. The SUPPOSER will extend the segment and label it.

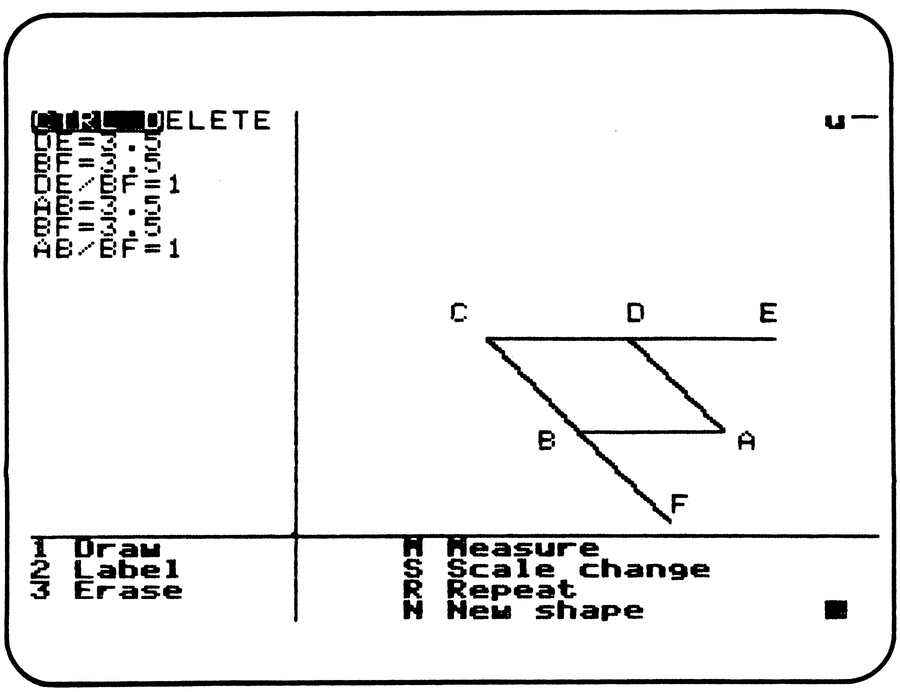
If instead you want to define the length of the extension in terms of some multiple of the unit length u, enter u for segment length and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw the extension and label it.

To illustrate the distinction between these two ways of specifying the length of an extension of a line segment, consider the following example and figures. Quadrilateral ABCD is a rhombus whose sides measure 3.5 and in which angle DAB = 60 degrees and angle ABC = 120 degrees. Suppose we extend side BC from point B by a length defined by AB * 1, and we also extend side CD from point D by a length of 3.5 units.

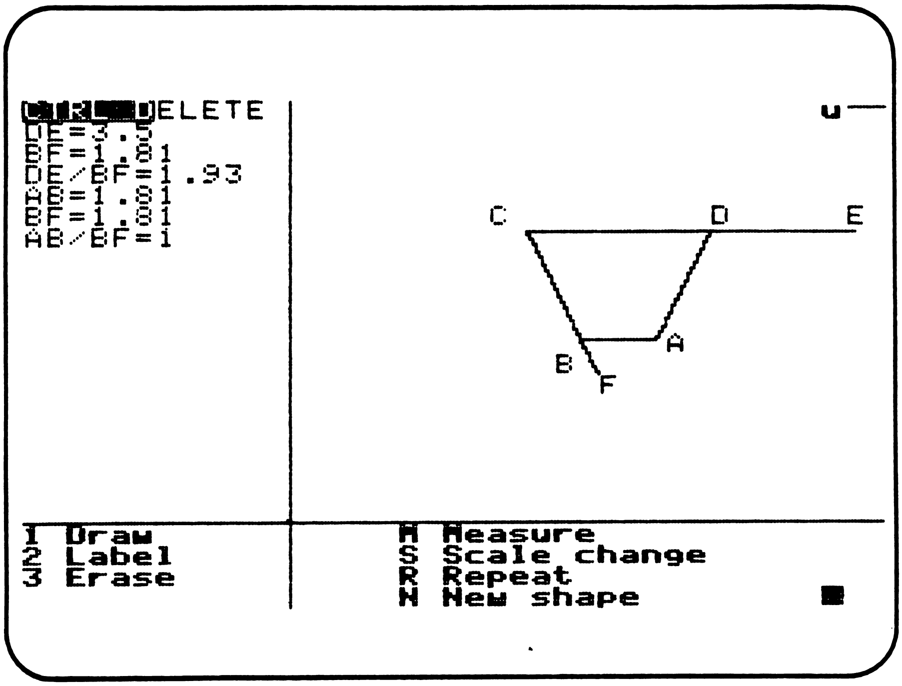
The length of the extension from B is defined in terms of the length of the segment AB. The length of the extension from D is defined in terms of a certain number of screen units. Even though the two extensions have the same numerical value, when we repeat the construction on a different quadrilateral or if we rescale the drawing, the two extensions will behave differently.

If the construction of the two extensions is repeated on another quadrilateral, the length of the extension from vertex B will remain equal to the length of the segment AB, while the length of the extension from vertex D is still 3.5 units.

Here are the extensions constructed on rhombus ABCD:



In this figure, the constructions have been repeated on a random trapezoid. Note the difference in the length of the extension drawn from B (with its length defined in terms of AB) when the construction is repeated.



The second way to specify the length of an extension, to intersect segment(s), allows you to draw the extension until it intersects the segment or segments you name. When the message reads

> Length defined by:
> 1 (segment or unit) * constant
> 2 to intersect segment(s)

enter the number 2 and then the name of the segment that you want the extension to intersect. Press RETURN and the SUPPOSER will draw the extension to intersect that segment. If you want the extension to intersect two segments, enter the name of the first segment, then the name of the second segment. The SUPPOSER will draw the extension to intersect the segments that you have named.

4 Angle bisector

Choice 4 under Draw is Angle bisector. Choosing this option allows you to draw the bisector of any angle. The SUPPOSER will ask for the name of the angle to be bisected. You will then be asked to specify the length of the angle bisector 1) in terms of a segment length or the unit length, or 2) to intersect segments on the screen.

Suppose you want to bisect angle ABC in quadrilateral ABCD with a bisector equal in length to AD. After entering the number 4, the message will read

```
Bisect angle:
```

asking you to enter the name of the angle you wish to bisect (ABC). Now the message will change to:

```
Length defined:
1 by (segment or unit) * constant
2 to intersect segment(s)
```

Enter the number 1 to define the length of the bisector by a length equal to segment AD. Now enter AD for segment or unit length. The message will now read:

```
Length defined:
   (segment or unit) * constant =
       AD           *
```

Now enter 1 for the constant and press RETURN. The SUPPOSER will extend the segment, label its endpoints, and return you to the MAIN MENU.

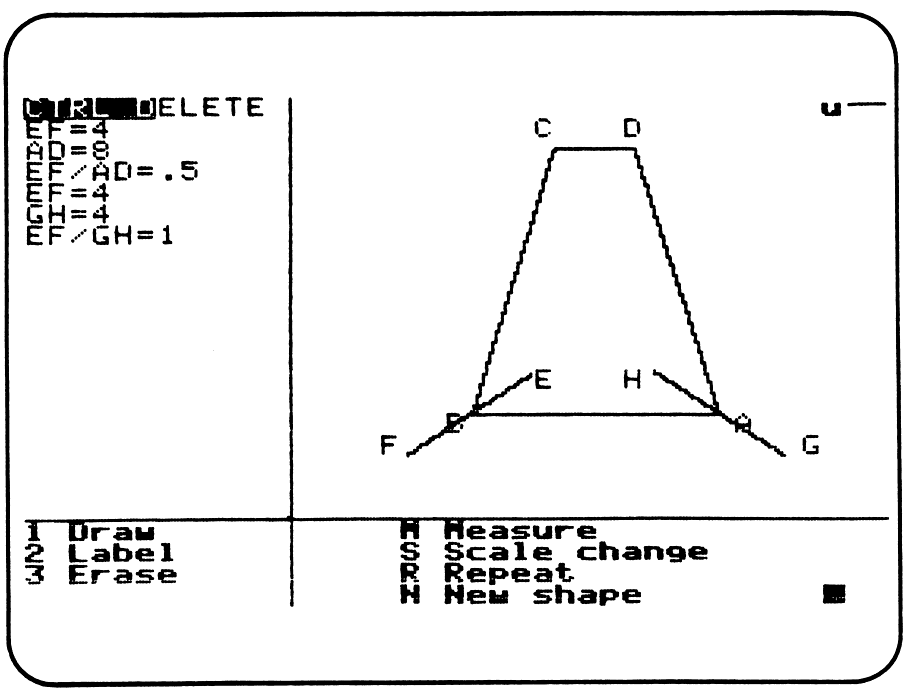
If instead you want to define the length of the bisector in terms of some multiple of the unit length u, enter u for segment length and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw the extension and label it.

To illustrate the distinction between these two ways of specifying the length of an angle bisector, consider the following example and figures. Quadrilateral ABCD is an isosceles trapezoid with side lengths AD = BC = 8. Suppose we draw the angle bisector of ABC and define its length as AD * .5, and we also draw the angle bisector of DAB with its length defined as 4 units.

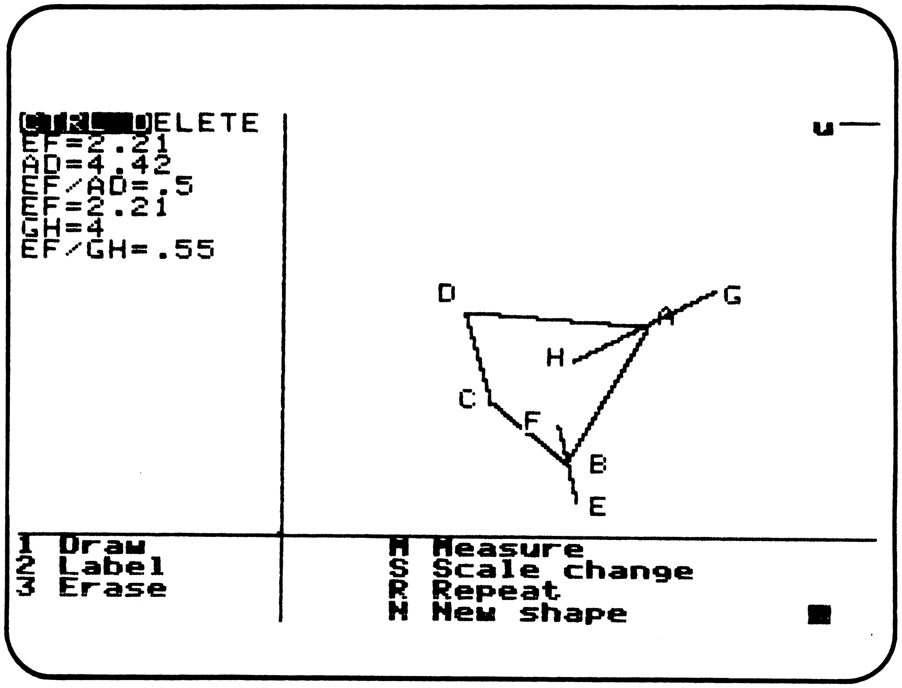
The length of the bisector through angle ABC is defined in terms of the length of the segment AD. The length of the bisector through DAB is defined in terms of a certain number of screen units. Even though the two bisectors have the same numerical value, when we repeat the construction on a different quadrilateral or if we rescale the drawing, the two bisectors will behave differently.

If the construction of the two bisectors is repeated on another quadrilateral, the length of the bisector through angle ABC will remain one half of the length of the segment AD, while the length of the bisector through angle DAB is still 4 units.

Here are two bisectors constructed on isosceles trapezoid ABCD:



In this figure, the constructions have been repeated on a kite. Note the difference in the length of the bisector through angle ABC (with its length defined in terms of AD) when the construction is repeated.



The second way to specify the length of an angle bisector, to intersect segment(s), allows you to draw the extension until it intersects the segment or segments you name. When the message reads

> Length defined by:
> 1 (segment or unit) * constant
> 2 to intersect segment(s)

enter the number 2 and then the name of the segment that you want the bisector to intersect. Press RETURN and the SUPPOSER will draw the extension to intersect that segment. If you want the bisector to intersect two segments, enter the name of the first segment, then the name of the second segment. The SUPPOSER will draw the bisector to intersect the segments that you have named.

5 Parallel

Choice 5 under Draw is Parallel. Choosing this option allows you to draw a line parallel to any line segment on the screen. The SUPPOSER will first ask through what point the parallel is to be drawn. It will then ask which segment it will parallel. You will then be asked to specify the length of the parallel 1) in terms of some segment or the unit length, or 2) to intersect a segment on the screen.

Suppose you want to draw a parallel through point E, parallel to AB, equal in length to AB, where E is the intersection of the diagonals AC and BD in parallelogram ABCD. After entering the number 5, Parallel, the message on the screen will read:

    Parallel through point:

Enter the name of the point through which you want the parallel to pass (E). The message will now read:

    Parallel through point: E
    Parallel to line:

Enter the name of the segment that you want to parallel (AB). The message on the screen will read:

    Length defined:
    1 by (segment or unit) * constant
    2 to intersect segment(s)

Enter 1 to define the length of the parallel by a segment or unit length. The message will now read:

    Length defined:
         (segment or unit) * constant

To construct a parallel with a length equal to segment AB, enter AB for segment or unit. The message will now read:

    Length defined:
         (segment or unit) * constant =
             AB   *

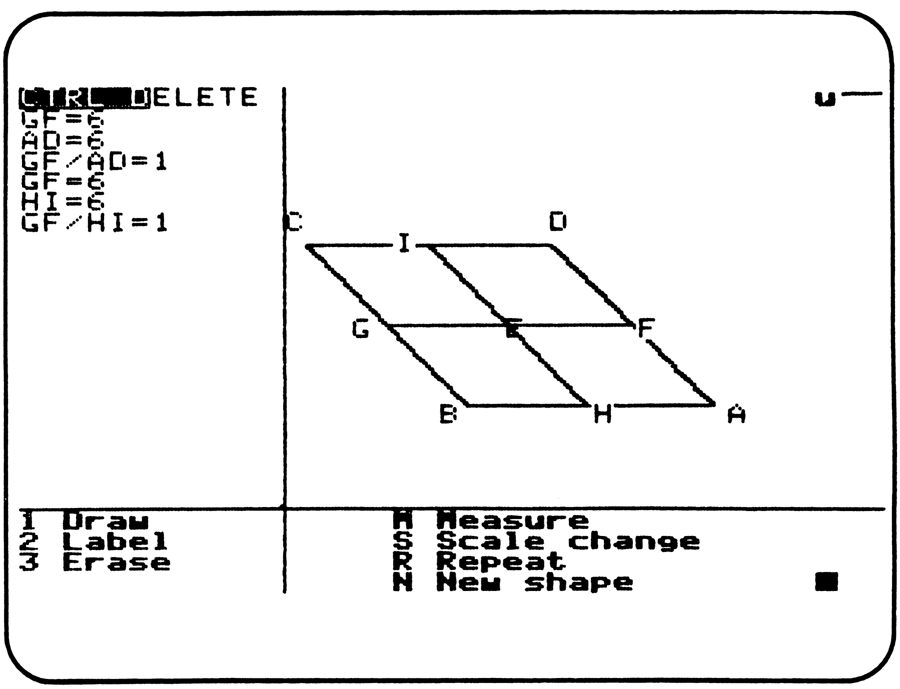
Now enter 1 for the constant and press RETURN. The SUPPOSER will draw the parallel, label the endpoints FG, and return you to the MAIN MENU.

If instead you want to define the parallel in terms of some multiple of the unit length u, enter u for segment or unit and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw and label the parallel.

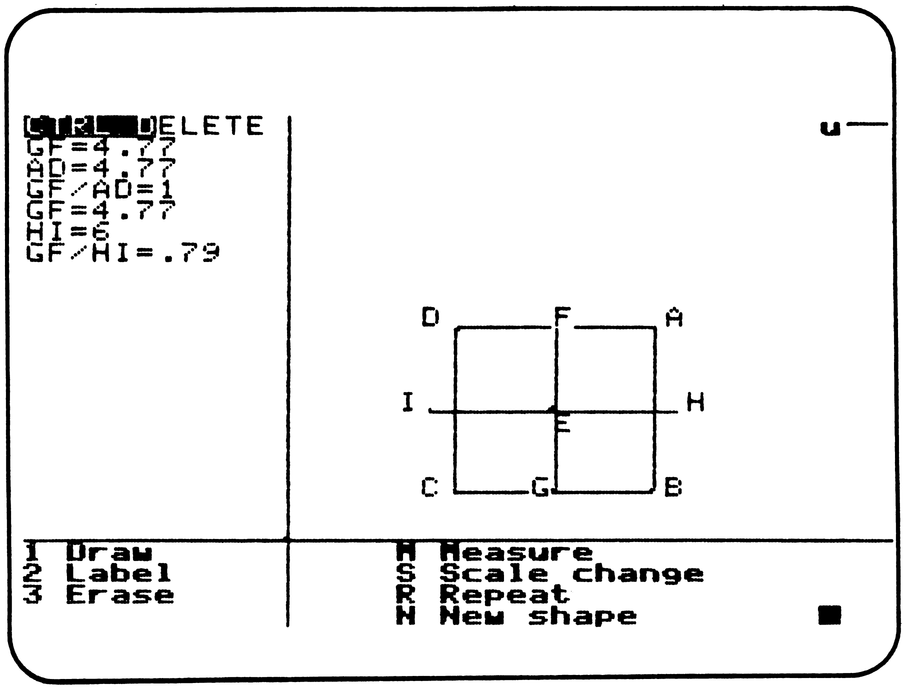
To illustrate the distinction between these two ways of specifying the length of a parallel, consider the following example and figures. Quadrilateral ABCD is a parallelogram with side lengths AD = BC = 6, with diagonals intersecting at point E. Suppose we draw a parallel through E, parallel to AB. Let the length of this parallel be AD * 1. Suppose, in addition, we draw another parallel through E, parallel to AD. Let the length of this parallel be 6 units long.

The length of the parallel drawn through E parallel to AB is defined in terms of the length of the segment AD. The length of the parallel drawn through E parallel to AD is defined in terms of a certain number of screen units. Even though the lengths of the two parallels have the same numerical value, when we repeat the construction on a different quadrilateral, or if we rescale the drawing, the two parallels will behave differently.

Here are the parallels drawn on parallelogram ABCD:



In this figure, the constructions have been repeated on a square. Note the difference in length of the parallel drawn through point E (defined in terms of segment AD) when the construction is repeated.



The second way to specify the length of the parallel, to intersect segment(s), allows you to extend a parallel until it intersects the line segment or segments you name. When the message on the screen reads

Length defined:
1 by (segment or unit) * constant
2 to intersect segment(s)

enter 2, then the name of the segment you want the parallel to intersect. Press RETURN and the SUPPOSER will draw the parallel to intersect that segment. If you wish to draw the parallel to intersect two segments, enter the name of the first segment, then enter the name of the second segment, and the SUPPOSER will draw the parallel to intersect the two segments that you have named and label the intersections.

6 Perpendicular

Choice 6 under Draw is Perpendicular. Choosing this option allows you to draw a line perpendicular to any line segment. The SUPPOSER will first ask through what point the perpendicular is to be drawn and then to which segment it will be perpendicular. You will then be asked to specify the length of the perpendicular 1) in terms of some segment or the unit length, or 2) to intersect segment(s) on the screen.

Suppose you want to draw a perpendicular through point E, perpendicular to side AB, with a length equal to side AB, where is E is the intersection of the two diagonals in rectangle ABCD whose sides are 5 and 8 units long. After entering 6, Perpendicular, the message on the screen will read:

```
Perpendicular through point:
```

Enter the name of the point through which you want the perpendicular to pass (E). The message will now read:

```
Perpendicular through point: E
Perpendicular to line:
```

Enter the name of the segment to which it will be perpendicular (AB). The message on the screen will now read:

```
Length defined:
1 by (segment or unit)  *  constant
2 to intersect segment(s)
```

Select option 1 to define the length of the perpendicular by a segment or the unit length. To construct a perpendicular with a length equal to segment AB, enter AB for segment or unit. The message will now read:

```
Length defined:
     (segment or unit)  * constant =
         AB             *
```

Now enter 1 for the constant and press RETURN. The SUPPOSER will draw the perpendicular, label the endpoints, and return you to the MAIN MENU.

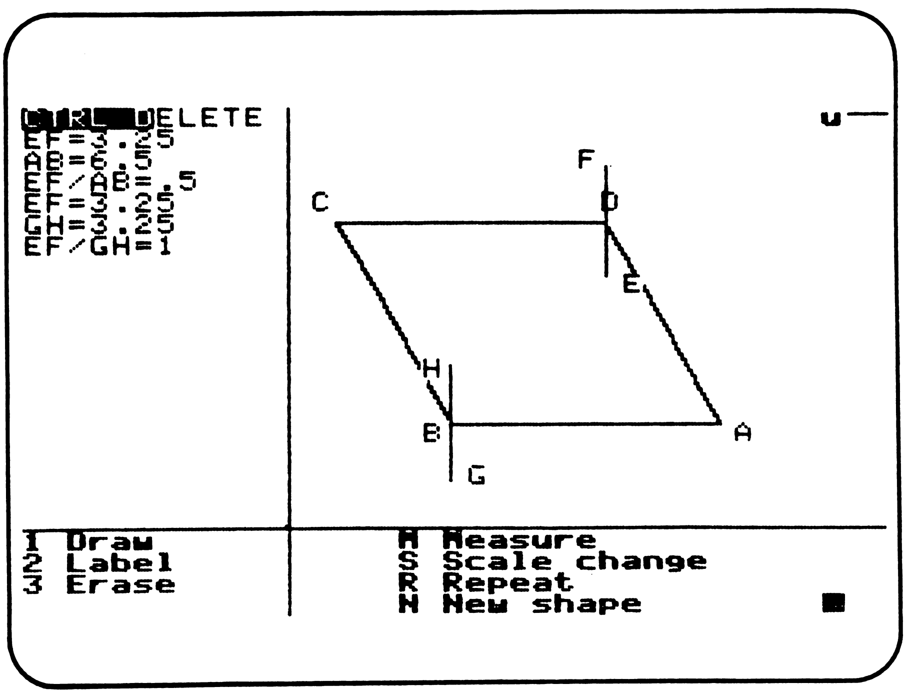
If instead you want to define the perpendicular in terms of some multiple of the unit length u, enter u for segment or unit and then for constant, enter the multiplier that you want to use. Press RETURN and the SUPPOSER will draw the perpendicular, label it, and return you to the MAIN MENU.

To illustrate the distinction between these two ways of specifying the length of a perpendicular, consider the following example and figures. Quadrilateral ABCD is a rhombus with side lengths equal to 6.5.

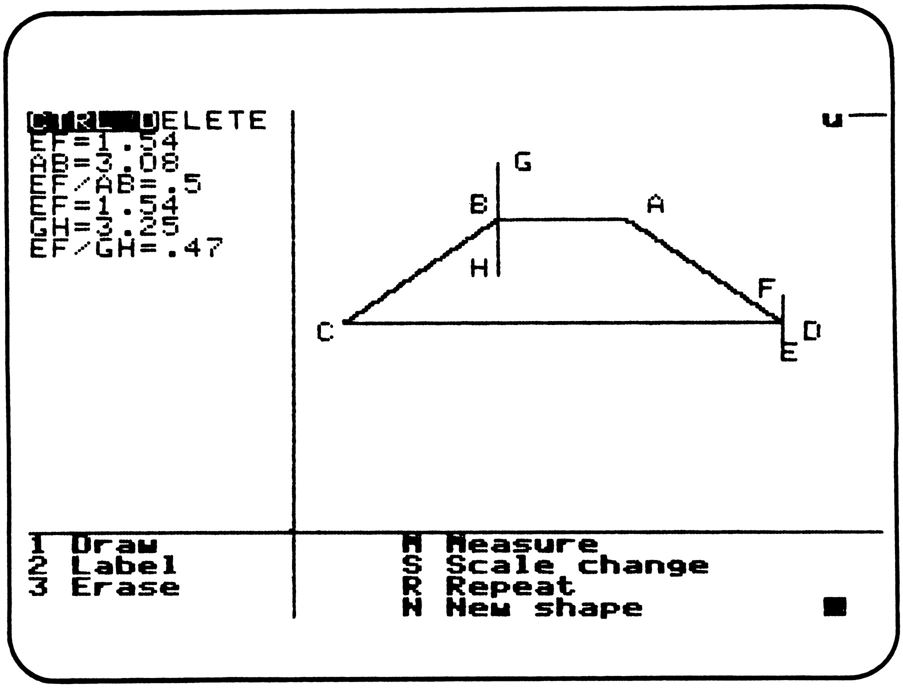
Suppose we draw a perpendicular through D, perpendicular to AB. Let the length of this perpendicular be AB * .5. Suppose, in addition, we draw another perpendicular through B, this one perpendicular to CD. Let the length of this perpendicular be 3.25 units long.

The length of the perpendicular drawn through D is defined in terms of the length of the segment AB. The length of the perpendicular drawn through B is defined in terms of a certain number of screen units. Even though the lengths of the two perpendiculars have the same numerical value, when we repeat the construction on a different quadrilateral, or if we rescale the drawing, the two perpendiculars will behave differently.

Here are the perpendiculars constructed on rhombus ABCD:



In this figure, the same constructions have been repeated on an isosceles trapezoid. Note the difference in length of the perpendicular drawn through D perpendicular to AB (defined in terms of the length of AB) when the construction is repeated.



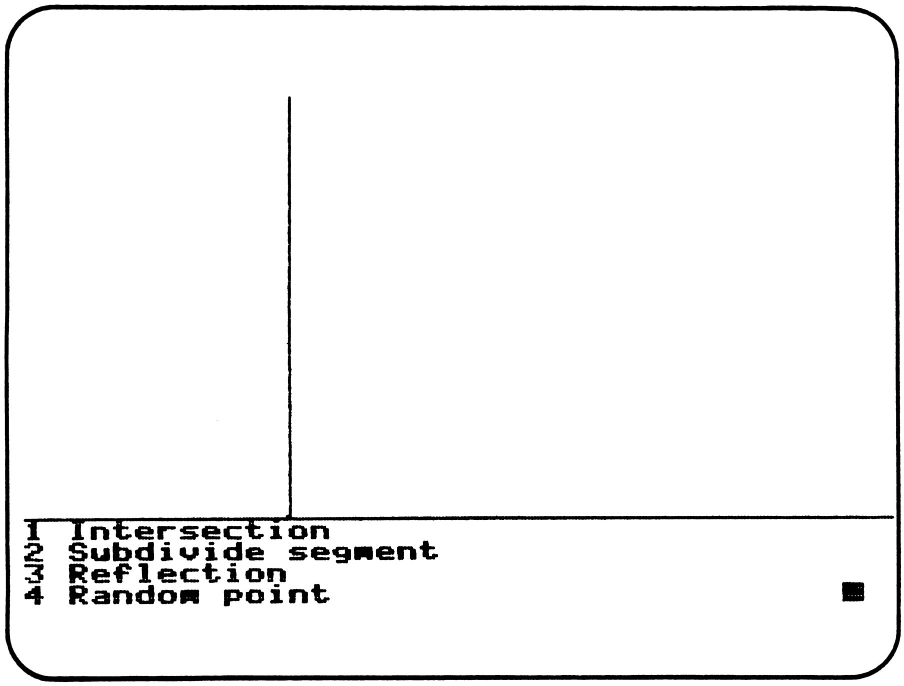
The second way to specify the length of the perpendicular, to intersect segment(s), allows you to extend a perpendicular until it intersects the line segment or segments you name. When the message on the screen reads

> Length defined:
> 1 by (segment or unit) * constant
> 2 to intersect segment(s)

enter 2, then the name of the segment you want the perpendicular to intersect. Press RETURN and the SUPPOSER will draw the perpendicular to intersect that segment. If you wish to draw the perpendicular to intersect two segments, enter the name of the first segment, then enter the name of the second segment. The SUPPOSER will draw the perpendicular to intersect the two segments that you have named, label them, and return you to the MAIN MENU.

LABEL

Choosing this option enables you to label various points on your quadrilateral or construction. After pressing L, you will see the following submenu screen:



1 Intersection

Choosing 1 Intersection allows you to label the intersection of two line segments. The SUPPOSER will ask you to specify the lines whose intersection you want to label.

Suppose you want to label the intersection of lines AC and BD. After entering the number 1, the message on the screen will read:

Intersection of line:

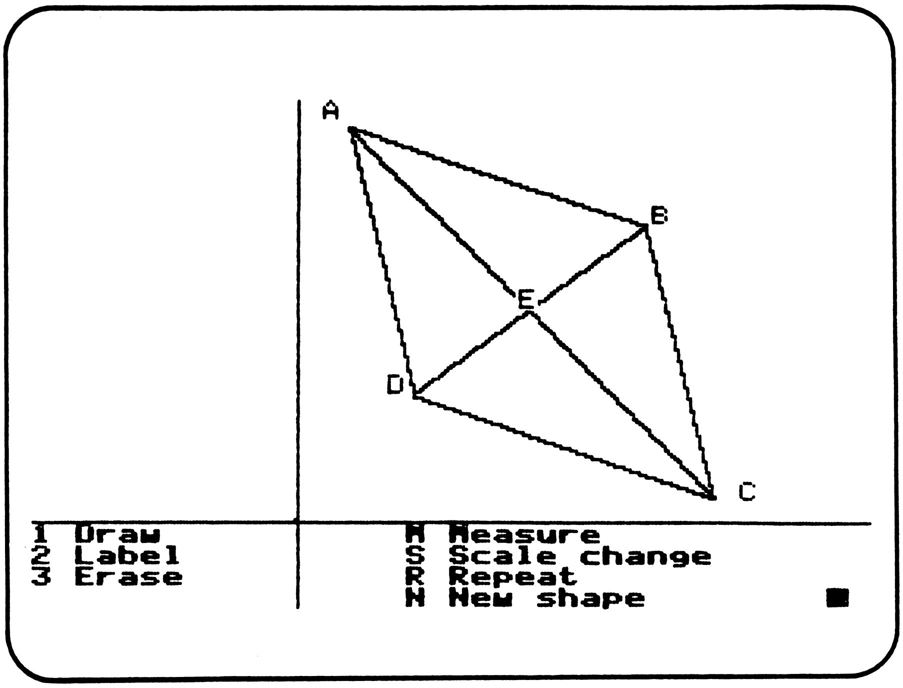
Enter the two letters that label the one line segment (AC) and the message will now read:

Intersection of line: AC
With line:

Now enter the two letters that label the other line (BD). The SUPPOSER will label the intersection and return you to the MAIN MENU.

There is no way to label the intersection of a circle and a line segment or the intersection of two circles.

Here, for example, is rhombus ABCD with its diagonals drawn. The intersection of the diagonals has been labeled.

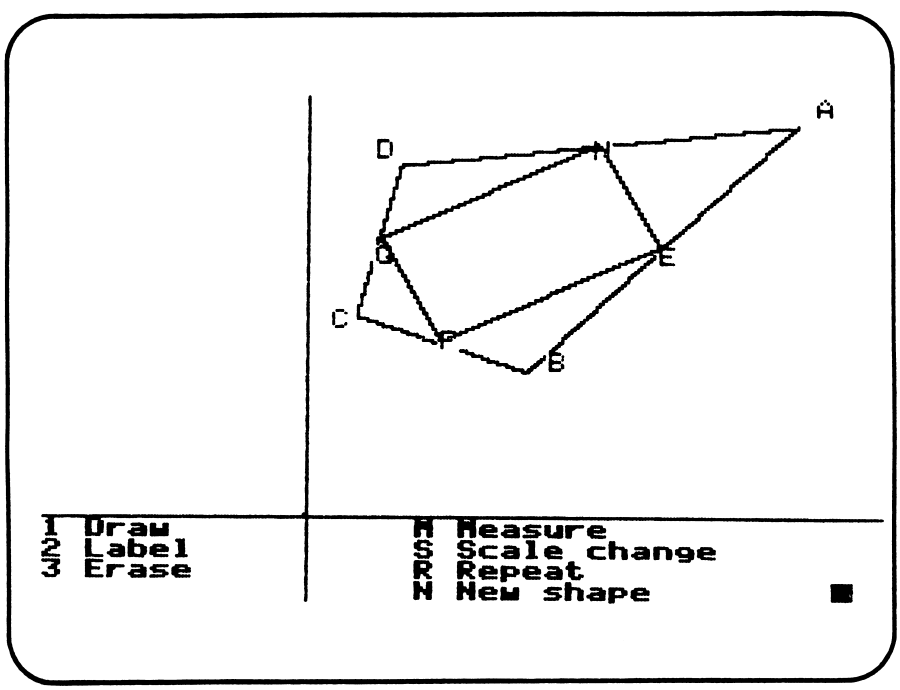


2 Subdivide segment

This choice allows you to subdivide any segment into any number of subdivisions. The SUPPOSER will ask you to identify the name of the segment to be subdivided and the number of subdivisions that you desire (the number of subdivisions may not exceed 8).

> Suppose you want to divide segment AC into five sections. After entering the number 2, the message on the screen will read:
>
> Subdivide segment:
>
> Enter the name of the segment you wish to subdivide (AC). The message will now read:
>
> Subdivide segment: AC
> Number of sections:
>
> The number of sections can range from 2 to 8. Enter the number of sections you desire (in this case 5), and the SUPPOSER will subdivide the segment, label the points of the subdivisions, and return you to the MAIN MENU.

Here, for example, is a quadrilateral with each of its sides divided into two segments. The lines connecting the points that subdivide the segments have been drawn.



3 Reflection

This choice allows you to reflect either a point or a line segment in any line. The SUPPOSER will ask you to identify the point or line that you want reflected and in which line you want it reflected.

Suppose you want to reflect a point (A) in a line (BC) on the screen. After entering the number 3, the message on the screen will read:

Reflection of:

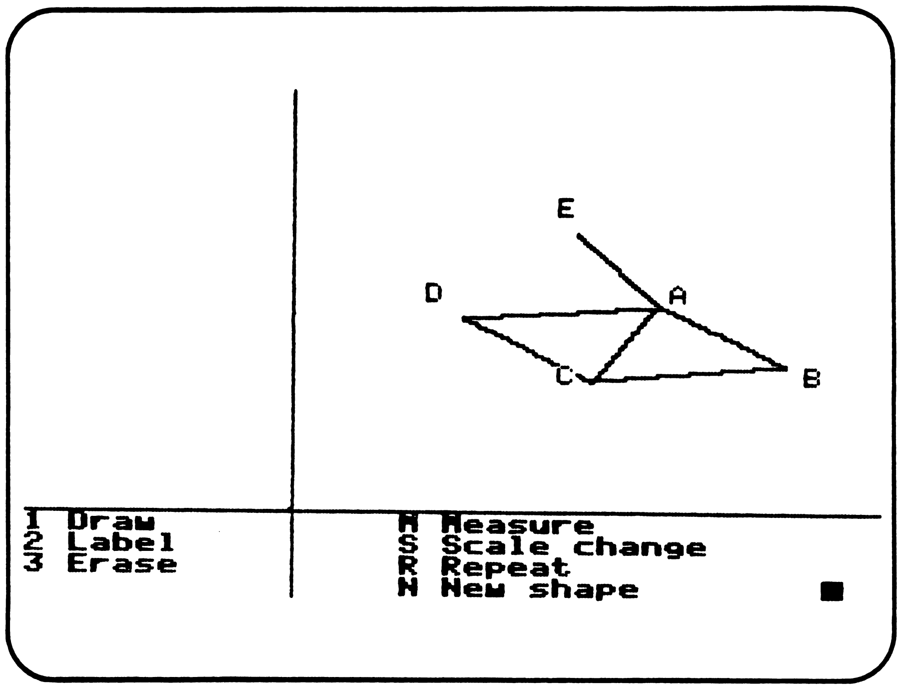
Enter the name of the point (A) and then press RETURN. After pressing RETURN, the message will read:

Reflection of: A   in:

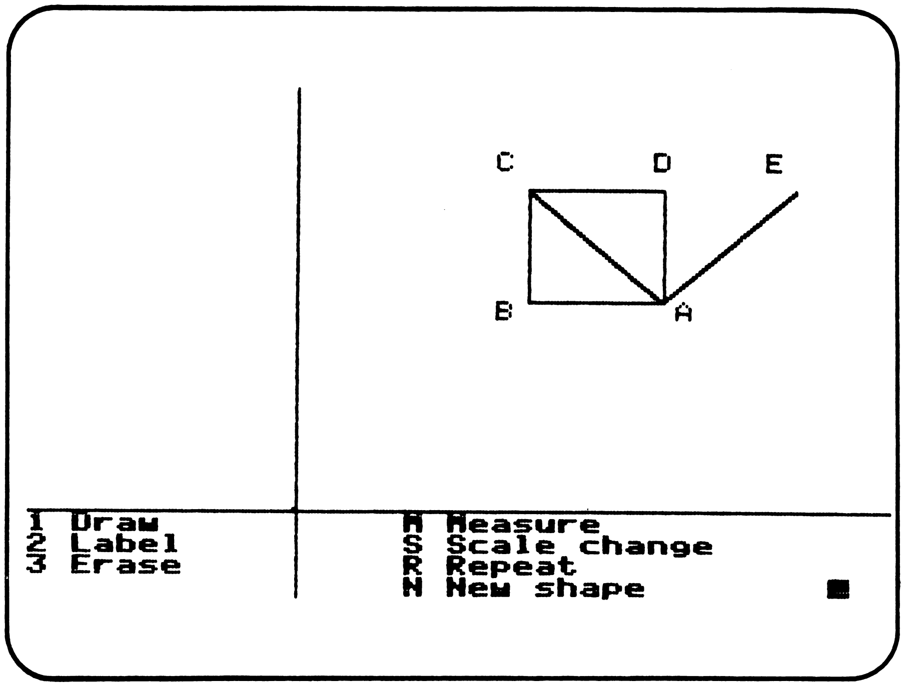
Now enter the name of the line segment in which you would like to reflect the point (BC). The SUPPOSER will locate and label the reflected point and return you to the MAIN MENU.

Suppose you want to reflect a line segment in another line segment. After entering the number 3, enter the name of the line segment you wish to reflect (you need not press RETURN), and the letters that label the line segment in which the reflection is to be carried out. The SUPPOSER will locate and label the reflected line segment and return you to the MAIN MENU.

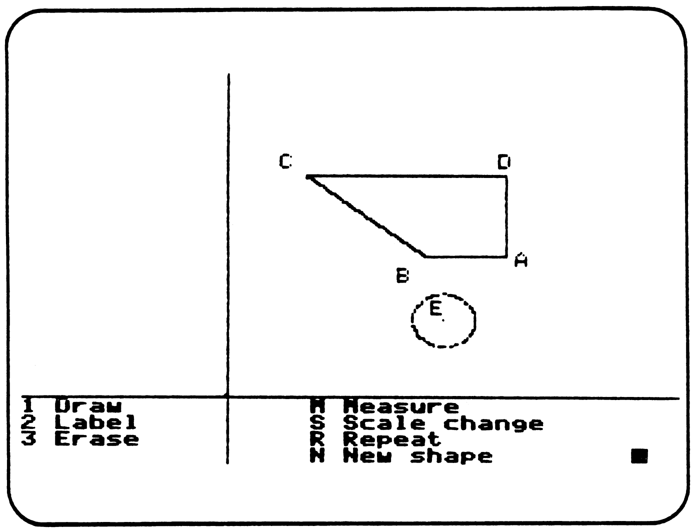
Here is a random parallelogram with one diagonal (AC) drawn. AC has been reflected in AD.



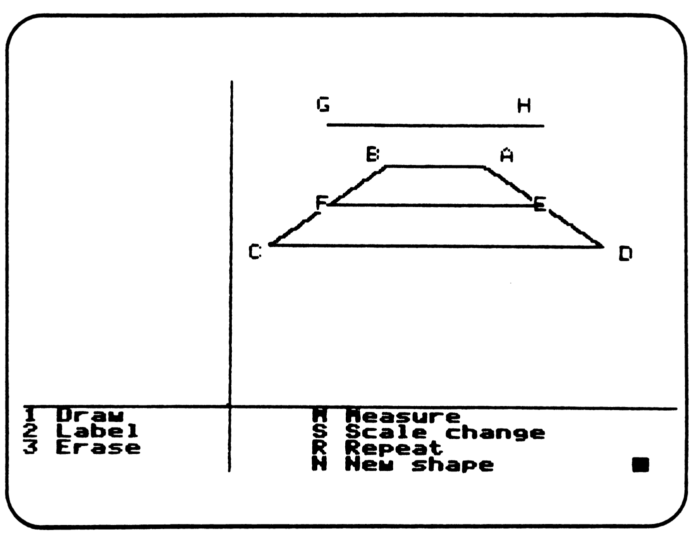
Here is the same construction repeated on a square:



In this construction, the point E (with a circle drawn about it for clarity) is the reflection of point A in line segment BC:



In this construction, segment GH is the reflection of segment EF in segment AB:



4 Random point

This option allows you to place a point at random on a line segment, inside a shape, or outside a shape. The SUPPOSER will ask you to identify on which segment, or inside or outside of which shape you wish a random point to be placed.

After entering the number 4, the message will read:

1 On segment:
2 Inside quad:
3 Outside quad:

Suppose you want to place a random point on a segment. Enter the number 1 and then enter the name of the segment on which you wish to place a point at random. The SUPPOSER will place the point on the segment and label it.

Suppose you want to place a random point inside a quadrilateral. Enter the number 2 and then enter the name of the shape in which you wish to place a point at random. The SUPPOSER will place the point inside the shape and label it.

Suppose you want to place a random point outside a quadrilateral. Enter the number 3 and then enter the name of the shape outside of which you wish to place a point at random. The SUPPOSER will place the point outside the shape and label it.

ERASE

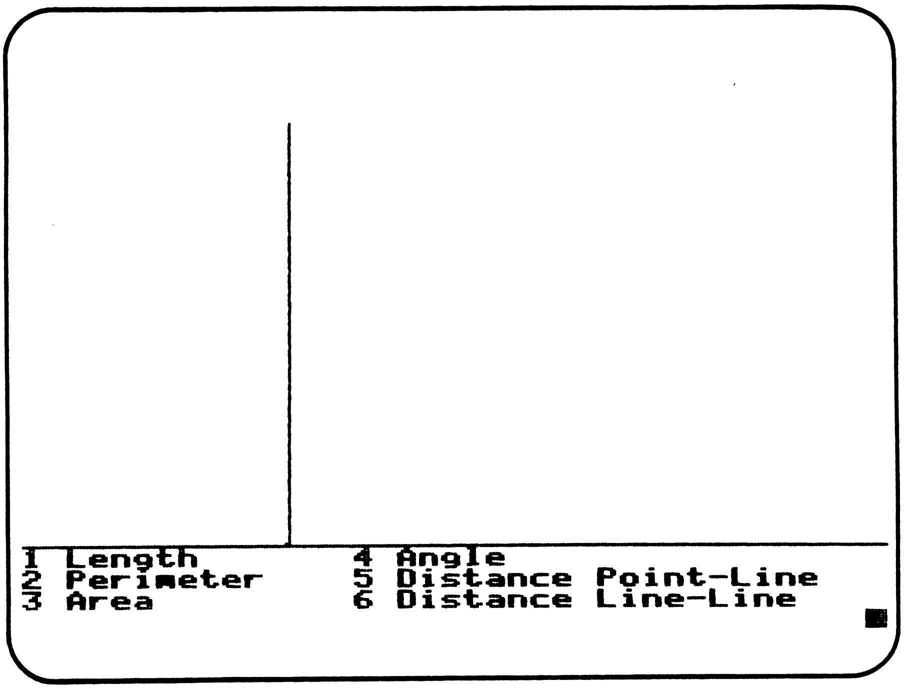
This option allows you to erase segments and to erase labels on the screen. The SUPPOSER will ask you which segment or label(s) you wish to erase.

> After entering 3 for Erase from the MAIN MENU, the message on the screen will read:
>
> 1 Erase segment:
> 2 Erase label(s):
>
> Suppose you wish to erase a segment. Enter the number 1 and then the name of the segment you wish to erase. The SUPPOSER will erase the segment and return you to the MAIN MENU.
>
> Suppose you want to make a diagram less cluttered by erasing some of the labels. Enter the number 2, then the name(s) of the label(s) you wish to erase, and press RETURN. The SUPPOSER will erase the label from the screen and return you to the MAIN MENU.

Even after being erased, the segment or the label exists in the memory. You can still refer to it even though it does not appear on the screen. For example, if you erase the segment GC or the labels G and C, you can still draw a circle centered on G, measure the distance GC, or draw a parallel to GC. Using the Erase option has no effect on the limits for labels and constructions.

MEASURE

The Measure option allows you to measure various components of your construction. Pressing M will result in the following submenu screen:



Measurements are accurate to within plus or minus 1% due to the limitations of the computer graphic display. The program may report, for example, an angle measure of 179 degrees rather than 180 degrees, or a length of 3.99 or 4.01 rather than 4. Is a ruler accurate to 1/400?

1 Length

Choice 1 under Measure is Length. This option allows you to measure the length of any segment on the screen. You may also measure the distance between labeled points on the screen that have no line segment drawn betweeen them. In addition, you can measure the sum, difference, product, and ratio of any two lengths and square any length. The SUPPOSER will ask you to identify the name of the segment whose length you wish to measure and will display the value of the length at the bottom of the screen and in the Data Column on the left of the screen. (The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new quadrilateral. You may also clear the column when the program is on one of the menus by entering CONTROL D.) All measurements are in terms of the unit length u, displayed in the upper right-hand corner of the screen.

Suppose you want to measure the length of side AB in quadrilateral ABCD. After entering the number 1, the message on the screen will read:

Length of segment:

Enter the name of the segment (AB) and press RETURN. The SUPPOSER will measure the segment and display the value at the bottom of the screen and in the Data Column. The same procedure can be used to measure the distance between any two labeled points on the screen that are not connected by a segment.

You now have two options: To continue to measure segment lengths, press the SPACE BAR; to return to the MAIN MENU, press ESCAPE.

Suppose you want to measure the ratio of two sides of a parallelogram, for example, the ratio of side AB to side CD. After entering the number 1, the screen will read:

Length of segment:

Enter the name of the first segment (AB) and the screen will read:

Length of segment: AB

+ - * / ^ or RETURN

Now press / (located on the same key as "?"). The screen will now read

Length of segment: AB = 5 /
Length of segment:

and the value (AB=5) will appear in the Data Column. Now enter the name of the second segment (CD) and press RETURN. The SUPPOSER will display the value of the second measure (CD=5) and give you the ratio of the two segments (AB/CD=1) at the bottom of the screen and in the Data Column. Now you can either continue to measure lengths by pressing the SPACE BAR, or return to the MAIN MENU by pressing ESCAPE.

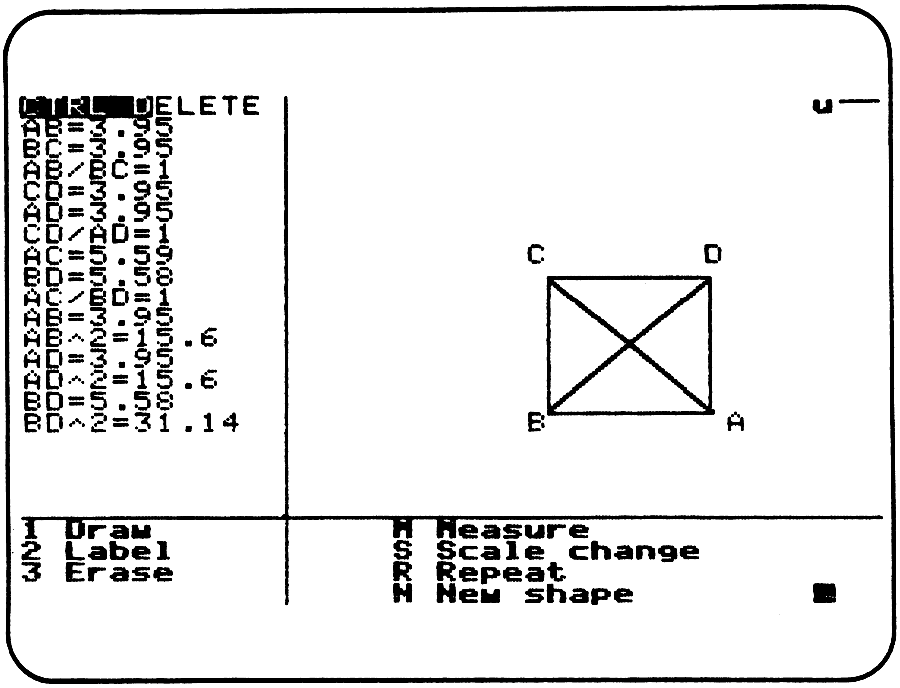
To measure the sum of two lengths, follow the same procedure, but press the + key rather than / after entering the name of the first segment.

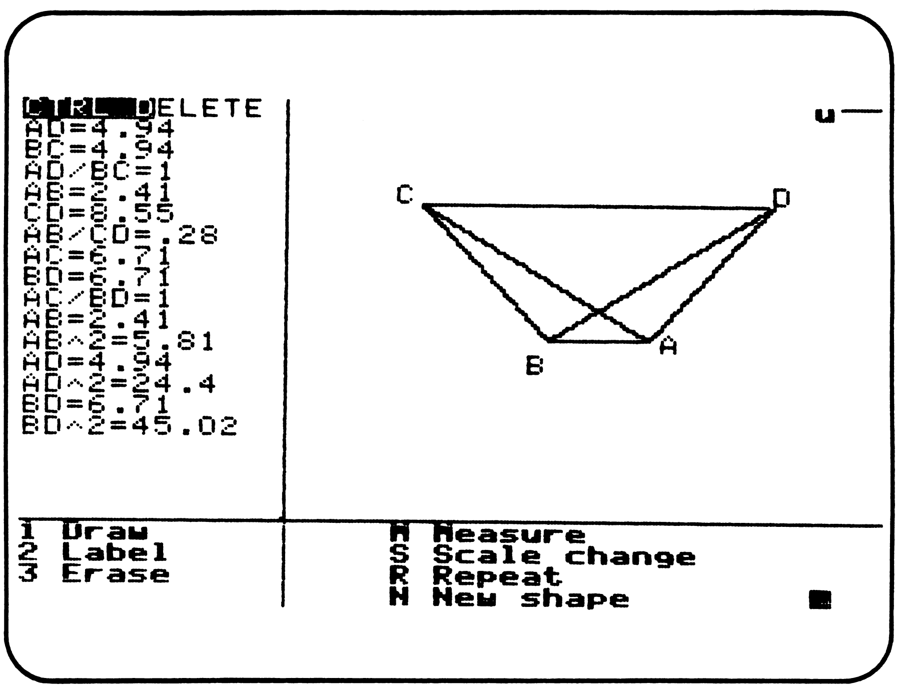
To measure the difference of two lengths, follow the same procedure, but press the - key rather than the / after entering the name of the first segment.

To measure the product of two lengths, follow the same procedure, but press the * key rather than the / after entering the name of the first segment.

To square the measure of any length, press the ^ key after you enter the name of the segment.

Here are the results of some length measurements on two quadrilaterals:





2 Perimeter

Choice 2 under Measure is Perimeter. This option allows you to measure the perimeter of any quadrilateral or triangle on the screen. You may also measure the perimeter of a quadrilateral that is defined by four labeled points or a triangle that is defined by three points, but is not drawn on the screen. In addition, you can measure the sum, difference, product, and ratio of any two perimeters (two quadrilaterals, two triangles, or a quadrilateral and a triangle) and square the measure of a perimeter. The SUPPOSER will ask you to specify the name of the quadrilateral or triangle whose perimeter you wish to measure and will display the value of the perimeter at the bottom of the screen and in the Data Column on the left of the screen. (The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new quadrilateral. You may also clear the column when the program is on one of the menus by entering CONTROL D.) All measurements are in terms of the unit length u, displayed in the upper right-hand corner of the screen.

Suppose you want to measure the perimeter of quadrilateral ABCD. After entering the number 2, the message on the screen will read:

Perimeter of shape:

Now enter the name of the quadrilateral (ABCD) whose perimeter you wish to measure and press RETURN. The SUPPOSER will measure the perimeter and display the value at the bottom of the screen and in the Data Column. The same procedure can be used to measure the perimeter of a quadrilateral defined by four points on the screen, but not drawn.

You now have two options: To continue to measure perimeters, press the SPACE BAR; to return to the MAIN MENU, press ESCAPE.

Suppose you want to measure the ratio of the perimeters of two quadrilaterals, ABCD and EFGH (created by subdividing each side of ABCD into two sections). After entering the number 2, the screen will read:

Perimeter of shape:

Enter the name of the first quadrilateral (ABCD) and the message will read:

> Perimeter of shape: ABCD
>
> \+ - * / ^ or RETURN

Now press / (located on the same key as "?"). The screen will now read

> Perimeter of shape: ABCD =27 /
> Perimeter of shape:

and the value (P:ABCD = 27) will appear in the Data Column. Now enter the name of the second quadrilateral (EFGH) and press RETURN. The SUPPOSER will display the value of the second perimeter (P:EFGH=16.6) and give you the ratio of the two perimeters (ABCD/EFGH=1.63) at the bottom of the screen and in the Data Column. Now you can either continue to measure perimeters by pressing the SPACE BAR, or return to the MAIN MENU by pressing ESCAPE.

To measure the sum of two perimeters, follow the same procedure, but press the + key rather than / after entering the name of the first shape.

To measure the difference between two perimeters, follow the same procedure, but press the - key rather than the / after entering the name of the first shape.

To measure the product of two perimeters, follow the same procedure, but press the * key rather than the / after entering the name of the first shape.

To square the measure of a perimeter, press the ^ key after entering the name of the shape.

To measure the perimeter of a triangle (ABC), enter the name of the triangle and press RETURN to indicate that the shape is defined by three letters. The message will now read

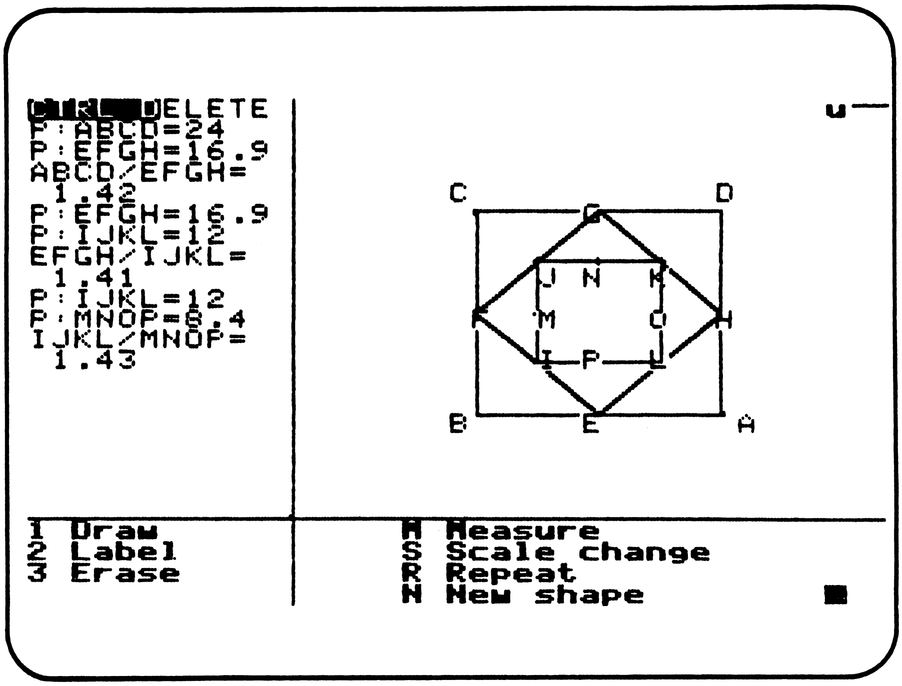
Perimeter of shape: ABC

+ - * / ^ or RETURN

asking you whether you want to carry out an operation or see the value of the perimeter of the triangle. To carry out an operation, follow the procedure described above for quadrilaterals (but press RETURN after naming the triangle(s) to indicate that the shape is defined by three letters). If you only want to measure the perimeter of triangle ABC, press RETURN.

You now have two options: To continue to measure perimeters, press the SPACE BAR; to return to the MAIN MENU, press ESCAPE.

Here are the results of some perimeter measurements on a set of squares:



3 Area

Choice 3 under Measure is Area. This option allows you to measure the area of any quadrilateral or triangle on the screen. You may also measure the area of a quadrilateral that is defined by four labeled points or a triangle defined by three points, but is not drawn on the screen. In addition, you can measure the sum, difference, product, and ratio of any two areas (two quadrilaterals, two triangles, or a quadrilateral and a triangle), and square an area. The SUPPOSER will ask you to name the quadrilateral or triangle whose area you wish to measure and will display the value of the area at the bottom of the screen and in the Data Column on the left of the screen. (The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new quadrilateral. You may also clear the column when the program is on one of the menus by entering CONTROL D.) All measurements are in terms of the unit area u, displayed in the upper right-hand corner of the screen.

Suppose you want to measure the area of quadrilateral ABCD. After entering the number 3, the message on the screen will read:

Area of shape:

Now enter the name of the quadrilateral whose area you want to measure (ABCD) and press RETURN. The SUPPOSER will measure the area and display the value at the bottom of the screen and in the Data Column. The same procedure can be used to measure the area of a quadrilateral defined by four points on the screen, but not drawn.

You now have two options: To continue measuring areas, press the SPACE BAR; to return to the MAIN MENU, press ESCAPE.

Suppose you want to measure the ratio of the areas of two quadrilaterals, ABCD and EFGH (created by subdividing each side of ABCD into two sections). After entering the number 3, the screen will read:

Area of shape:

Enter the name of the the first quadrilateral (ABCD) and the screen will read:

Area of shape: ABCD

+ - * / ^ or RETURN

Now press / (located on the same key as "?"). The screen will read

Area of shape: ABCD = 23.46 /
Area of shape:

and the value (A:ABCD=23.46) will appear in the Data Column. Now enter the name of the second quadrilateral (EFGH) and press RETURN. The SUPPOSER will display the value of the second area (A:EFGH =11.73) and give you the ratio of the two areas (ABCD/EFGH =2) at the bottom of the screen and in the Data Column. Now you can either continue to measure areas by pressing the SPACE BAR, or return to the MAIN MENU by pressing ESCAPE.

To measure the sum of two areas, follow the same procedure, but press the + key rather than / after entering the name of the first area.

To measure the difference between two areas, follow the same procedure, but press the - key rather than the / after entering the name of the first area.

To measure the product of two areas, follow the same procedure, but press the * key rather than the / after entering the name of the first area.

To square an area, press the ^ key after entering the name of the quadrilateral.

To measure the area of a triangle (ABC), enter the name of the triangle and press RETURN to indicate that the shape is defined by three letters. The message will now read

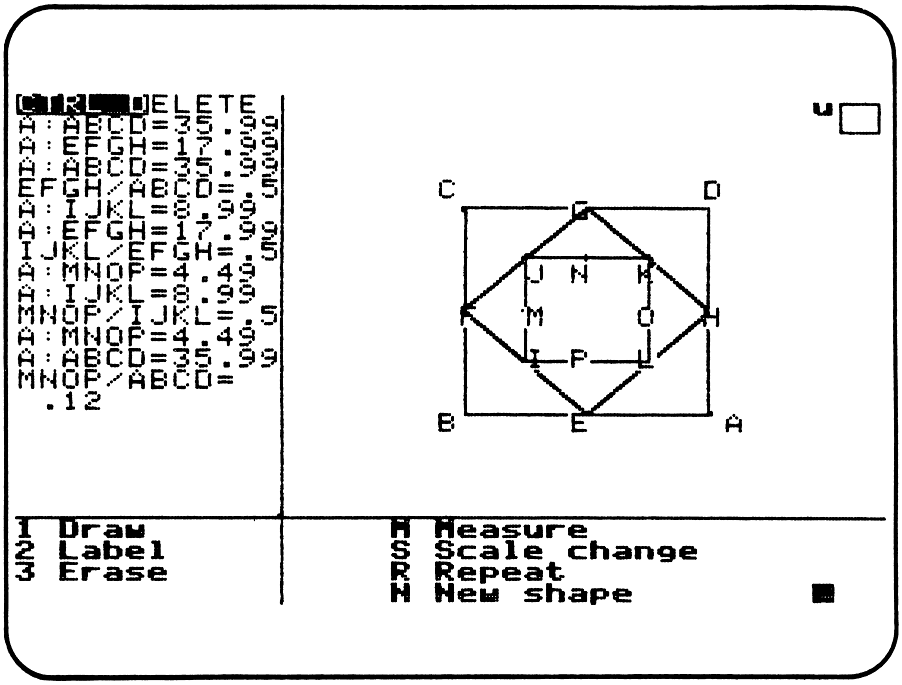
Area of shape: ABC

+ - * / ^ or RETURN

asking you whether you want to carry out an operation or see the value of the area of the triangle.
To carry out an operation, follow the procedure described above for quadrilaterals (but press RETURN after naming the triangle(s) to indicate that the shape is defined by three letters). If you only want to measure the area of triangle ABC, press RETURN.

You now have two options: To continue to measure areas, press the SPACE BAR; to return to the MAIN MENU, press ESCAPE.

Here are the results of some area measurements on a set of squares:



4 Angle

Choice 4 under Measure is Angle. This option allows you to measure any angle, in degrees, on the screen. You may also measure an angle that is defined by three labeled points but is not drawn on the screen. You can measure the sum, difference, product, and ratio of any two angles and square the measure of an angle as well. The SUPPOSER will ask you to name the angle you wish to measure and will display the value of the angle at the bottom of the screen and in the Data Column on the left of the screen. (The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new quadrilateral. You may also clear the column when the program is on one of the menus by entering CONTROL D.)

Suppose you want to measure the angle ABC in parallelogram ABCD. After entering the number 4, the message on the screen will read:

Angle name (3 letters):

Now enter the name of the angle you want to measure (ABC) and press RETURN. The SUPPOSER will measure the angle and display the value at the bottom of the screen and in the Data Column. The same procedure can be used to measure an angle defined by three points on the screen, but not drawn.

You now have two options: To continue to measure angles, press the SPACE BAR; to return to the MAIN MENU, press ESCAPE.

Suppose you want to measure the sum of two angles, angle ABC and angle CDA, in parallelogram ABCD cited above. After entering the number 4, the screen will read:

Angle name (3 letters):

Enter the letters that label the first angle (ABC) and the screen will read:

Angle name (3 letters): ABC

+ - * / ^ or RETURN

Now press / (located on the same key as "?"). The screen will now read

Angle name (3 letters): ABC =149
Angle name (3 letters):

and the value (<ABC= 149) will appear in the Data Column. Now enter the name of the second angle (CDA) and press RETURN. The SUPPOSER will display the value of the second angle (<CDA= 149) and give you the ratio of the two angles (ABC/CDA= 1) at the bottom of the screen and in the Data Column. Now you can either continue to measure angles by pressing the SPACE BAR, or return to the MAIN MENU by pressing ESCAPE.

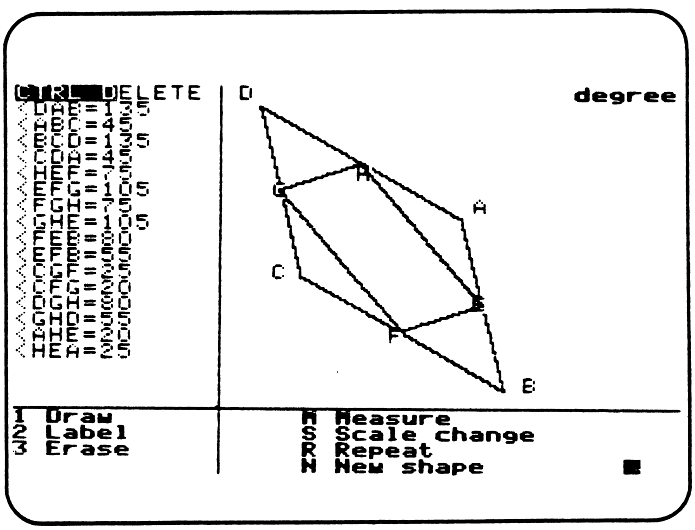
To measure the sum of two angles, follow the same procedure, but press the + key rather than / after entering the name of the first angle.

To measure the difference between two angles, follow the same procedure, but press the - key rather than the / after entering the name of the first angle.

To measure the product of two angles, follow the same procedure, but press the * key rather than the / after entering the name of the first angle.

To square the measurement, press the ^ key after entering the name of the angle.

Here, for example, are the results of some angle measurements on two parallelograms:



5 Distance Point-Line

Option 5 under Measure is Distance Point-Line. This option allows you to measure the perpendicular distance from any labeled point on the screen to any line segment on the screen. You may also measure the distance from a point to a line segment defined by two points but not drawn. You can measure the sum, difference, product, and ratio of such distances and square the distance as well. The SUPPOSER will ask you to name the point and the line segment that define the distance you wish to measure. All measurements are in terms of the unit length u, displayed in the upper right-hand corner of the screen.

The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new quadrilateral. You may also clear the column when the program is on one of the menus by entering CONTROL D.

Suppose you want to measure the perpendicular distance from a random point E, located outside of quadrilateral ABCD, to segment AB. After entering the number 5, the message on the screen will read:

Distance of point:

Enter the name of the point (E) and the message will now read:

Distance of point: E to line:

Now enter the name of the segment (AB) and press RETURN. The SUPPOSER will measure the perpendicular distance and display the value at the bottom of the screen and in the Data Column.

You now have two options: To continue to measure point-line distances, press the SPACE BAR; to return to the MAIN MENU, press ESCAPE.

Suppose you want to measure the ratio of two such distances, the ratio of the perpendicular distance of point A to line CD to the perpendicular distance of point C to line AD, where ABCD is a parallelogram. After entering the number 5, the message will read:

Distance of point:

Enter the name of the first point (A) and the message will now read:

Distance of point: A to line:

Enter the name of the first segment (CD) and the message will read:

Distance of point: A to line: CD

+ - * / ^ or RETURN

Now press / (located on the same key as "?") and the message will read

Distance of point: A to line CD = 9.25 /
Distance of point:

displaying the value of the first perpendicular distance at the bottom of the screen and in the Data Column. Now enter the name of the point (C) and the name of the segment (AD) of the second distance that you want to measure. The SUPPOSER will display the value of the second distance and give you the ratio of the distances at the bottom of the screen and in the Data Column. Now you can either continue to measure point-line distances by pressing the SPACE BAR, or return to the MAIN MENU by pressing RETURN.

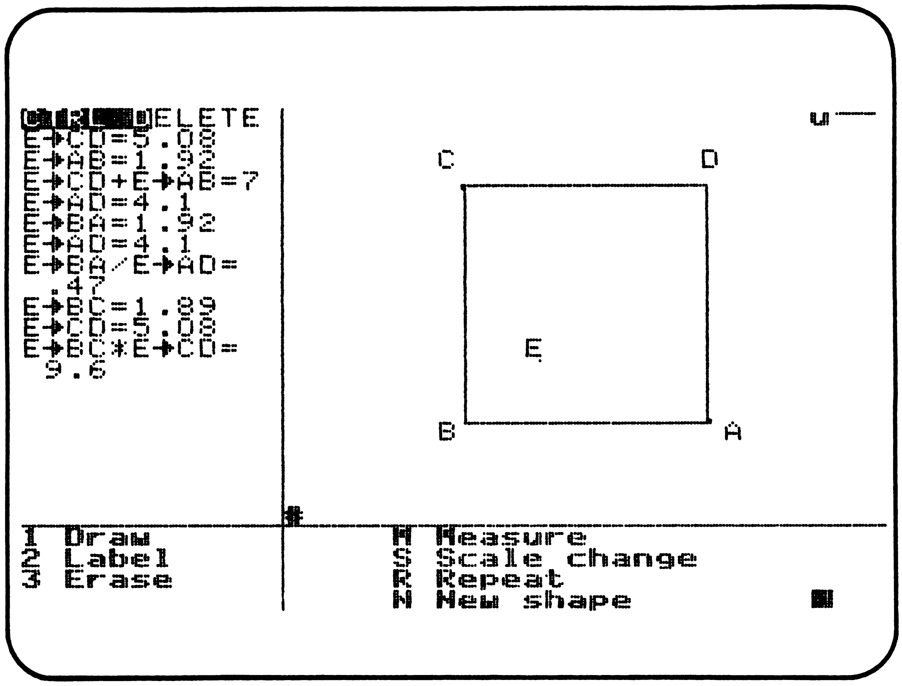
To measure the sum of two such distances, follow the same procedure, but press the + key rather than / after entering the names of the point and segment for the first distance.

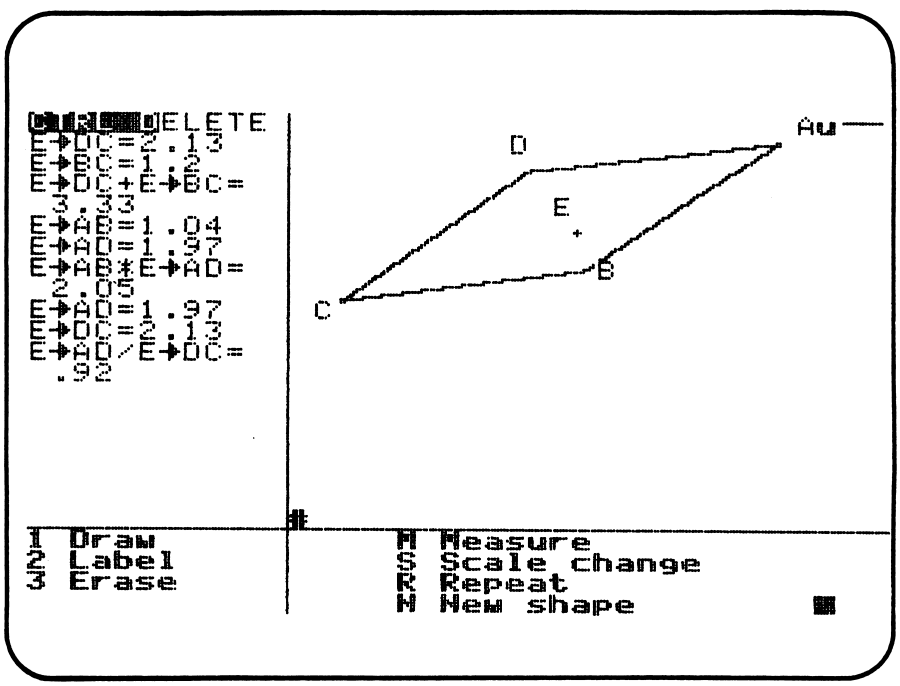
To measure the difference between two such distances, follow the same procedure, but press the - key rather than / after entering the names of the point and segment for the first distance.

To measure the product of two such distances, follow the same procedure, but press the * key rather than / after entering the names of the point and segment for the first distance.

To square such a distance, press the ^ after entering the names of the point and the segment that define the distance.

Here are the results of a set of measurements of point-line distances involving a random point in two parallelograms:





6 Distance Line-Line

Option 6 under Measure is Distance Line-Line. This option allows you to measure the perpendicular distance between any two parallel line segments on the screen. You may also measure the distance between parallel line segments that are labeled but not drawn. You can measure the sum, difference, product, and ratio of such distances and square such a distance as well. The SUPPOSER will ask you to name the letters that label the two segments (lines) whose distance you wish to measure. All measurements are in terms of the unit length u, displayed in the upper right-hand corner of the screen.

If you ask the SUPPOSER to measure the distance between two non-parallel or intersecting line segments, the message on the screen will read

Intersecting lines

since the perpendicular distance between non-parallel or intersecting lines cannot be defined.

The Data Column will display all your measurements, will scroll when it is full, and will clear when you create a new quadrilateral. You may also clear the column when the program is on one of the menus by entering CONTROL D.

> Suppose you want to measure the perpendicular distance from segment EF to side AB, where EF is the segment connecting the midpoints of AD and BC in trapezoid ABCD. After entering the number 6, the message will read:
>
> **Distance of line:**
>
> Enter the name of the first segment (EF) and the message will now read:
>
> **Distance of line: EF to line:**
>
> Now enter the name of the second segment (AB) and press RETURN. The SUPPOSER will measure the perpendicular distance and display the value at the bottom of the screen and in the Data Column.
>
> You now have two options: To continue to measure line-line distances, press the SPACE BAR; to return to the MAIN MENU, press ESCAPE.

Suppose you want to measure the ratio of two such distances, the ratio of the perpendicular distance of segment AB to line CD to the perpendicular distance of line EF to line CD in the trapezoid cited above. After entering the number 6, the message will read:

Distance of line:

Enter the name of the first segment (AB) and the message will now read:

Distance of line: AB to line:

Enter the name of the second segment (CD) and the message will read:

Distance of line: AB to line: CD

+ - * / ^ or RETURN

Now press / (located on the same key as "?") and the message will read

Distance of line: AB to line: CD = 6.5 /
Distance of line:

displaying the value of the first perpendicular distance at the bottom of the screen and in the Data Column. Now enter the name of the segments that define the second perpendicular distance (EF and CD). The SUPPOSER will display the value of the second distance and give you the ratio of the distances at the bottom of the screen and in the Data Column. Now you can either continue to measure line-line distances by pressing the SPACE BAR, or return to the MAIN MENU by pressing RETURN.

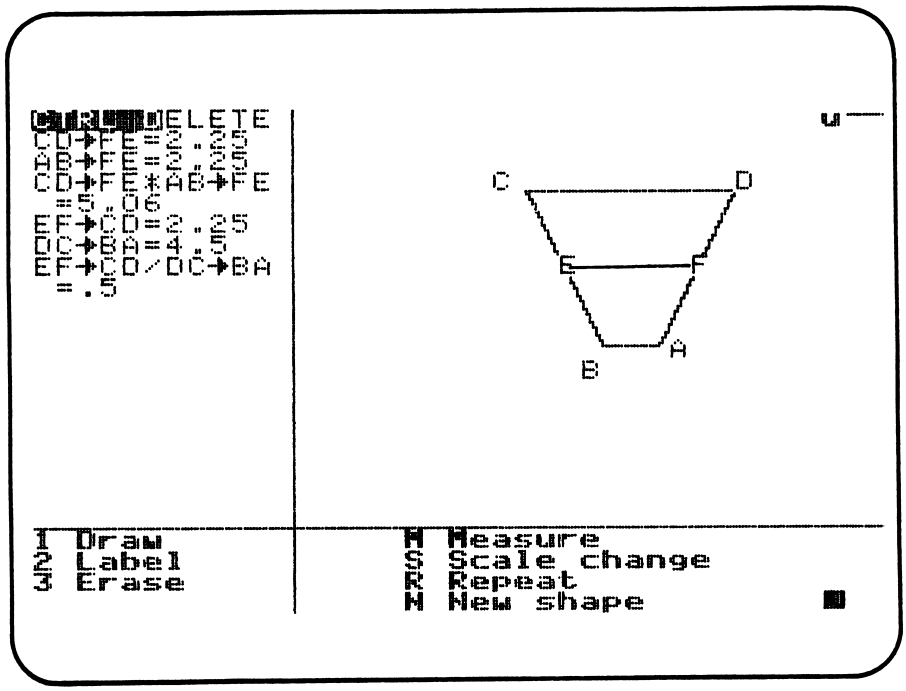
To measure the sum of two such distances, follow the same procedure, but press the + key rather than / after entering the names of the segments in the first distance.

To measure the difference between two such distances, follow the same procedure, but press the - key rather than / after entering the names of the segments in the first distance.

To measure the product of two such distances, follow the same procedure, but press the * key rather than / after entering the names of the segments in the first distance.

To square the measure of such a distance, press the ^ key after entering the names of the segments that define the distance.

Here are the results of a set of measurements of perpendicular distances between lines in a trapezoid:



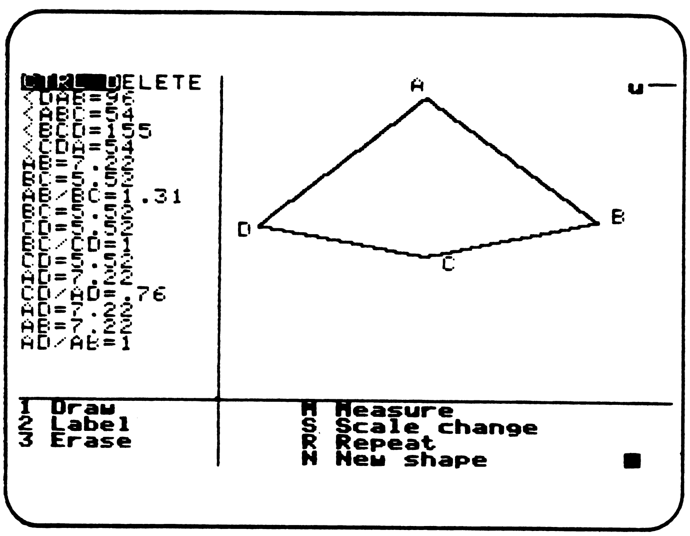
SCALE CHANGE

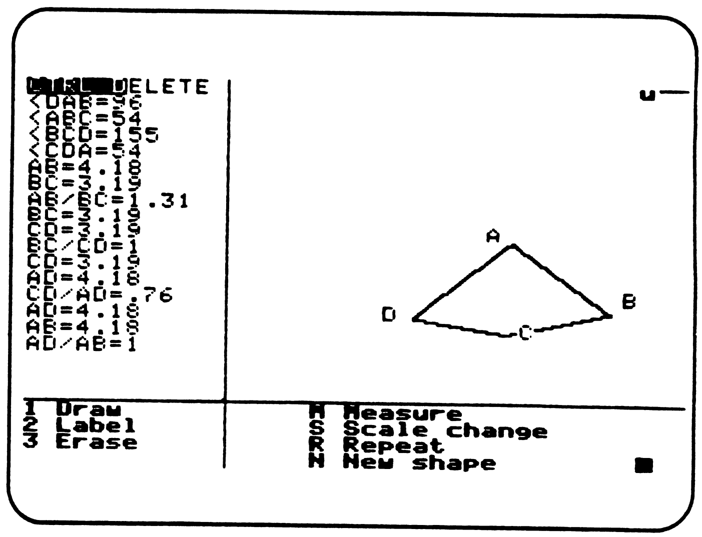
Any drawing on the screen can be drawn in one of two sizes. Choosing this option will cause the SUPPOSER to redraw the drawing on the screen in the OTHER size.

The option has another function as well. If a construction cannot be fully displayed on the screen and the beep sounds, try using the SCALE CHANGE option. In addition to changing the scale of the quadrilateral, SCALE CHANGE checks the placement of the quadrilateral on the screen. In most cases, the program can relocate the quadrilateral so that the construction can be displayed.

> To carry out a scale change, enter the letter S and the SUPPOSER will rescale the quadrilateral.

Here is quadrilateral ABCD and the same quadrilateral rescaled. Note the differences and similarities in the measurements on the two quadrilaterals.





REPEAT

The Repeat option allows you to repeat the construction you have just made on a new quadrilateral or on any one of the three most recent previous quadrilaterals.

To choose the REPEAT option in the MAIN MENU, press R and the the message on the screen will read:

Repeat construction:
1 on new shape
2 on previous shape

If you choose option 1, on new shape, you will be able to see the construction you have made on the screen carried out on a different quadrilateral. After entering 1, the SUPPOSER will return to the quadrilateral selection menu (described in detail in Section II.2 above). Follow the procedures for constructing a new shape. When the new shape is drawn, the message will read:

Press space bar to repeat construction.

Each time you press the SPACE BAR, a single step in the construction will be repeated. Continue to press the SPACE BAR until all steps in the construction have been replicated. When all the steps in the construction have been carried out, the SUPPOSER will return you to the MAIN MENU.

If you choose option 2, on previous shape, you will be able to repeat the construction you have made carried out on a quadrilateral that you worked with earlier.

After entering the number 2, the message will read:

<== ==> to see shape:
1 2 3 4
Press RETURN to select

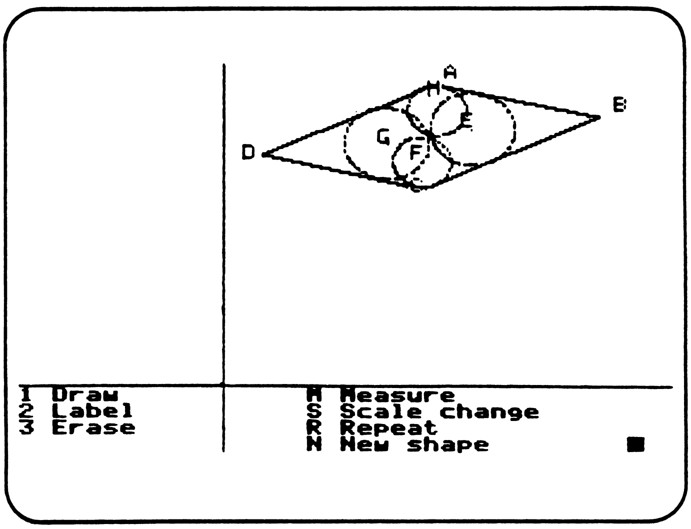
The program stores the four most recently constructed quadrilaterals and you can use the arrows to move back and forth among the four. As you move from number to number, each quadrilateral will appear on the screen and its corresponding number will be highlighted. When you have identified the quadrilateral on which you would like to see your construction repeated, press RETURN. The message will now read:

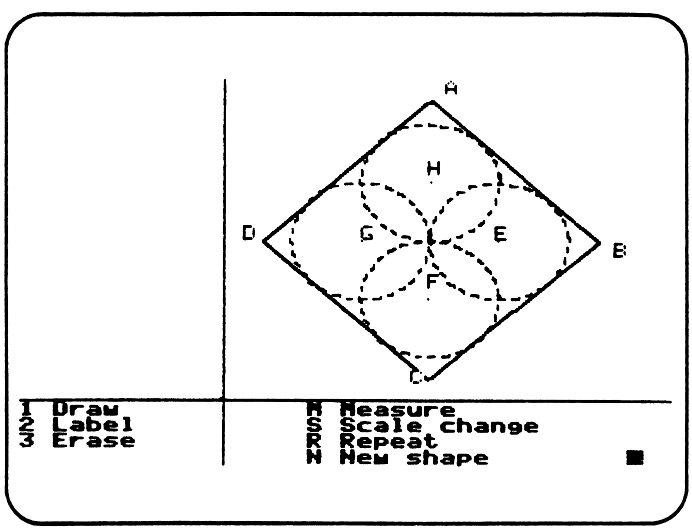
Press space bar to repeat construction.

Repeat

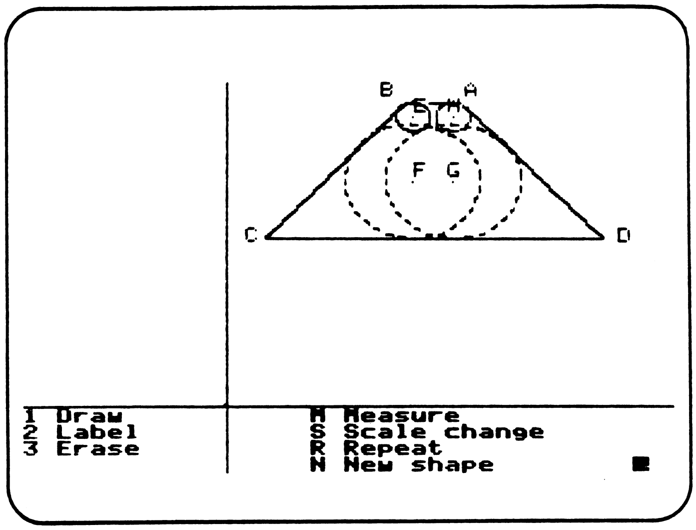
Each time you press the SPACE BAR, a single step in the construction will be repeated. Continue to press the SPACE BAR until all the steps in the construction have been replicated. When all the steps in the construction have been carried out, the SUPPOSER will return you to the MAIN MENU.

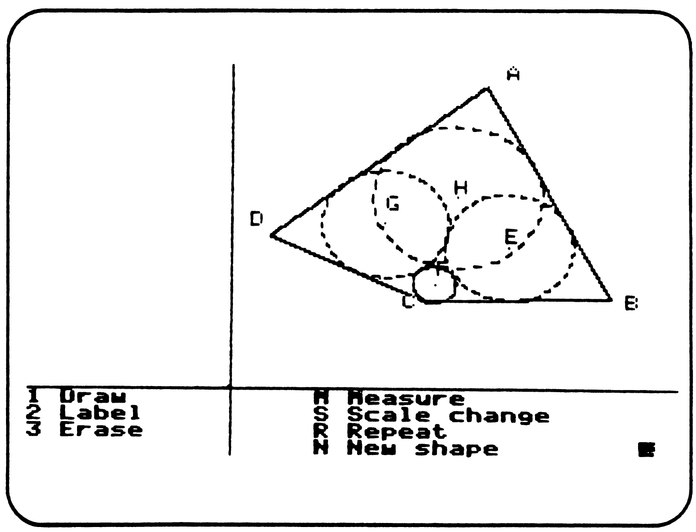
Here, for example, is a construction in which inscribed circles are drawn in four triangles (ABC, BCD, CDA, DAB) of parallelogram ABCD. This construction has been repeated on a square, an isosceles trapezoid, a kite, and a right angle trapezoid.

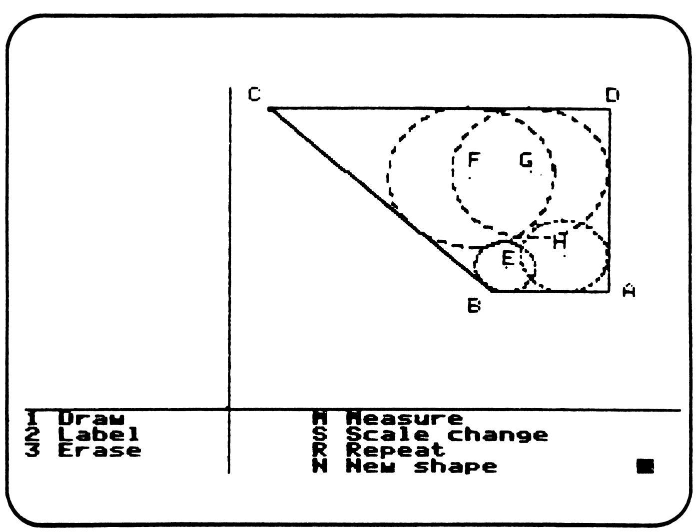




Repeat


B
A
E
H
F
G
C
D
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New shape


A
D
G
H
E
F
C
B
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New shape


C
D
F
G
H
E
B
A
1 Draw
2 Label
3 Erase
M Measure
S Scale change
R Repeat
N New shape

NEW SHAPE

The New Shape option allows you to select a randomly generated quadrilateral or subclass of quadrilateral, or to create your own quadrilateral on which to carry out constructions.

To select this option, enter the letter N. The screen will clear and the message will read:

1 Parallelogram	4 Quads/Circles
2 Trapezoid	5 Your own
3 Kite	

To choose one of these options for drawing a shape, enter the appropriate number.

For a detailed description on the operation of this option, see Section I.2, How Do I Make A Quadrilateral?

III. USING THE GEOMETRIC SUPPOSER AS A TEACHING TOOL

Quadrilaterals are a rich domain for geometry instruction, yet most geometry texts never exploit their potential. Texts tend to deal with but a small subset of the full range of four-sided polygons. And with this small subset, the level of inquiry rarely goes beyond a consideration of definitions and area. In fact, in many cases quadrilaterals are treated merely as vehicles for confirming and reinforcing theorems and concepts related to triangles.

Why is this the case? Several possible explanations come to mind. First, triangles are the fundamental building blocks of all polygons and therefore are given more attention and favored status over quadrilaterals. Also, given the diversity and the number of types of quadrilaterals and the competition among subjects for limited instructional time, there may simply not be enough class hours to give quadrilaterals their due.
Because of the diversity of quadrilaterals, there are many fewer "universal" theorems that hold true for all quadrilaterals than there are for triangles. Finally, the difficulty of working with straightedge and compass limits the number of shapes and examples with which a class can work and, as a result, limits the scope of the curriculum.

THE GEOMETRIC SUPPOSER: Quadrilaterals, however, is a tool which can turn these limiting conditions to advantage, can open new avenues for inquiry, and can deepen students' understanding of the properties and behaviors of all polygons. With its capacities to generate endless numbers of shapes, to measure, and to repeat constructions--with speed and ease, the SUPPOSER enables students and teachers to deal with a rich collection of topics and problems and to go beyond the standard curriculum if they so choose.

Using the SUPPOSER's options for generating shapes, making constructions, making measurements, and repeating constructions, teachers can pursue a variety of approaches to quadrilaterals. Exploration can focus on a particular subset of four-sided polygons, e.g, concentrating on parallelograms using the Measure, Draw, and Repeat options. Teachers can ask students to discover and to define the properties of quadrilaterals, gathering data from the shapes generated by the SUPPOSER and then confirming their conjectures using the option to create your own shapes by either the Sides and Angles method or the Diagonals method. They can emphasize the comparison of properties of triangles and quadrilaterals, using THE GEOMETRIC SUPPOSER: Quadrilaterals program by itself, or by using it in parallel with THE GEOMETRIC SUPPOSER: Triangles. Or, they can transform problems and theorems from textbooks into open-ended exercises. For example, in a standard text, a theorem is stated "The diagonals in a rhombus bisect the opposite angles"; with the SUPPOSER, however, one can ask "What are the properties of the diagonals in a rhombus? Which properties are true for all rhombuses and which are true for specific rhombuses? Which properties are common to all parallelograms?"

Before dealing with these and other strategies and approaches in some depth, this section first provides some general information about using the SUPPOSER in the classroom: how to think about the SUPPOSER in terms of access to hardware, how to help students work with the SUPPOSER, and how to prepare problems to use with the SUPPOSER.

To set the tone, here are some entries from the diary of Richard Houde, a mathematics teacher in the Weston (MA) High School who used THE GEOMETRIC SUPPOSER with his students during the 1983-84 school year. Although this entry deals with THE GEOMETRIC SUPPOSER: Triangles program, the ideas and the spirit apply equally to quadrilaterals.

> "This year I made the SUPPOSER a formal part of my two semester geometry classes; so much, in fact, that I did not distribute geometry textbooks to my students. The SUPPOSER, worksheets, and I became their textbook. I wrote problems for the SUPPOSER that asked students to state conjectures about geometric relationships. These conjectures actually represented many of the theorems one might find in a traditional high school geometry textbook. I also wrote problems for the SUPPOSER that extended simple geometric ideas to more complex ones and again asked students to state conjectures.
>
> "...I believe that the problems I posed to students required them to think. Students had to:
>
> (1) understand the statement of a given problem before they could begin to investigate it,
> (2) collect and organize data,
> (3) develop, guess, and check procedures for analyzing data,
> (4) exhibit counterexamples to show that conjectures were false,
> (5) write proofs for conjectures that they believed were true, and
> (6) organize and prepare written reports.
>
> "...Students worked independently on these problems both during and outside class time and occasionally asked me for hints or to clarify problem statements. When students found that they could not draw any conclusions regarding a given problem, they would group together to discuss it. These conversations often resulted in a restatement of the problem and the students went back to work on it again. When a group of students met to share their results, it was also not uncommon for them to argue whether or not their conjectures were true or false. In fact, I found some of the best problems for

the course while listening to these conversations or trying to answer their questions in class. These times became the most exciting in the course for me because we were all creating our own mathematics together. During one particular class, when I mentioned that perhaps it was about time to start distributing a textbook, one student responded, 'It's much more fun this way (without a textbook) because we are coming up with geometry ourselves.' Surprisingly, to me at least, the rest of the students nodded their heads in agreement."

Using THE GEOMETRIC SUPPOSER as part of a geometry class transforms learning geometry into making geometry. With this transformation, the balance shifts from proof to conjecture, from solutions to questions, from seeking answers to encouraging inquiry and investigation. For students, this means greater responsibility since the course of learning relies heavily on the findings of students and on the reactions of their classmates. For teachers, this may require some flexibility and patience in their efforts to provoke and to guide productive exploration.

The key to effective classwork with THE GEOMETRIC SUPPOSER is discussion. To create an environment conducive to rich and lively discussion several conditions need to be met:

-- There should be an interesting subject to discuss.

-- In order to participate and to contribute, every student should have knowledge of the subject under discussion, gained from direct experience exploring the subject either individually or in a group.

-- The atmosphere in the classroom should be one that welcomes discussion of any plausible idea, and one in which every student believes that such ideas are proper subjects for class discussion.

A. TEACHING WITH THE SUPPOSER

With A Single Computer

When there is only one computer in the classroom, the SUPPOSER is a useful tool for introducing new concepts, evoking ideas, answering questions, and exploring problems with the class as a whole. It is useful to have a large monitor in the front of the room or several small monitors around the room so that everyone in the class can see the display.

A teacher can use problems to provide the class with a collective challenge and manage the discussion so that students see themselves as part of a team trying to solve a puzzle. This kind of discussion can provide students with a model for devising and testing hypotheses and can lay the foundation for theorems and proofs.

Present a problem to the class, rather than teach a theorem. Generate a list of ideas. Solicit, or when appropriate, provide confirming examples or counterexamples. Use the SUPPOSER in lieu of asking students "to go to the blackboard." Ask students to assert conjectures and then subject them to the scrutiny of the class. Ask students to think about properties for similar shapes (e.g., different types of trapezoids) and then about whether these properties hold for other shapes as well.

Such discussions can occur at any time, but are more likely to be productive when a new topic or subject is introduced. Use the SUPPOSER to introduce new geometric phenomena and to stimulate investigation.

With Individual or Small Group Access to a Computer

When students can work on a computer on their own, a key role for the teacher during class time is to help students analyze their data and findings. The teacher becomes a partner in the conversation, helping students focus their arguments and look for evidence in their data. The process of students reporting to their classmates can be, and often is, an occasion for sharply differing opinions about some geometric phenomenon.

We have seen teachers use lab time effectively in two different ways:

- -- having students work on written problems in the lab on their own, outside of class time, for two or three hours per week time; or
- -- holding class in the lab one or two times per week and using the remaining class time to introduce new topics and discuss findings from the lab.

To guide student work, written problem sets and assignments are essential. The level of structure and guidance that you provide will depend on your assessment of your students.

After students have worked on a problem on their own, you can start the discussion by taking a student's findings and putting them on the blackboard. Ask the class if they found similar data and what they make of the data. Ask students to be explicit about their approaches: how they thought about the problem, why they chose to to make a particular construction or measurement on a specific quadrilateral. See if students took different approaches. Challenge them with examples that they have not considered. Ask if they see patterns emerging from the data.

At first these arguments are likely to be informal. But as students become more comfortable and nimble with the subject matter and the program, be clear that a written formal proof is expected.

Students reporting back to the class about the results of their inquiries in the lab provides a wonderful opportunity for teachers to gauge the level and degree of students' understanding. Further, it is a fine source of suggestions and ideas for further class exploration and discussion. As exciting as such discussion may be, there will be a point at which the teacher may wish to stop the general discussion and ask the students to write their conclusions or to devise proofs of what they believe.

Discussions of this sort are likely to be richer in content and vigor after the students have had an opportunity to explore some issue thoroughly. On the other hand, there can be brief discussions resolving the uncertainties of what some students call "strange drawings".

Lab work with the SUPPOSER is most successful when students work in pairs. With a sufficient number of computers, students can work individually, but because of the open-ended nature of the inquiry, the educational experience is likely to be richer when students can put their heads together and are forced to subject their ideas and actions to another's scrutiny. In cases where there are too few computers for students to work in pairs, we advise finding a way to schedule smaller groups of students.

Getting Students Started and Working Productively

To help students learn how to use THE GEOMETRIC SUPPOSER, demonstrate the use of the program to them with some simple problems drawn from your course materials. Here is an example of a simple problem that will help students learn about the SUPPOSER:

> Problem: What are the properties of and relationships among the triangles created by two diagonals in a quadrilateral?
>
> With the SUPPOSER: Draw a rectangle.
> Draw the two diagonals.
> Label the intersection of the diagonals.
> Measure the lengths, areas, perimeters, and angles of the four triangles.
> Repeat the construction and the measurements on a parallelogram.
> Repeat the procedures on other quadrilaterals.
> Use the scale option on some examples and and repeat the measurements.

From students and teachers, we've gathered some helpful hints about how to minimize discomfort and maximize learning in using the SUPPOSER:

-- To help students overcome the common fears of making mistakes or losing information, emphasize the following:

> You can always cancel your most recent key press by pressing ESC.
>
> At any time, the program remembers the four most recent shapes on which you have been working and can repeat your construction on any of them.
>
> Keep a written record of what you've done. Include such information as the steps you took to make a particular construction as well as all the measurements you made on that drawing.

-- In order to help the students record drawings, measurements and procedures, the teacher should give particular attention to the layout of the written assignment sheets. Make liberal use of tables to be filled in and blank space for drawings to be recorded.

-- Teach the use of the Repeat option in the SUPPOSER as early as possible. In addition to offering the students an opportunity to explore their ideas and conjectures, the Repeat option allows the students to remake constructions with greater ease and efficiency.

-- Suggest that students adopt the following techniques and strategies in using the SUPPOSER to explore problems:

Look at many examples;

Take notes;

Look at the data, make guesses that may be suggested by your data, and explore them in other cases;

Share ideas, difficulties, and interesting findings with your classmates or your partners in the lab.

Preparing Problems and Written Materials for Use with the SUPPOSER

With the SUPPOSER, students can draw, study, and solve most geometry problems in a traditional high-school geometry text-book. But this approach doesn't really exploit the instructional potential of the SUPPOSER because problems in traditional texts tend not to be open-ended, and are frequently not well suited to exploration. It is, however, often possible to start with problems presented in a traditional text and to change the statement of the problem so that it becomes an open-ended question that calls for research and investigation. In working with teachers, we've seen them create three different types of problems to use with the SUPPOSER at different points in the curriculum and with students of varying abilities. Here are examples of the three types of problems and how they relate to problems in a traditional text:

Defining a Concept or Proving a Theorem

In the textbook: Each diagonal in a rhombus bisects a pair of opposite angles.

With the SUPPOSER: What are the properties of diagonals in a rhombus? In all rhombuses? Which properties hold true for all rhombuses and for specific rhombuses? Which properties are common to all other parallelograms?

Establishing True or False Statements

In the textbook: Prove that the median in a trapezoid is equal to one-half the sum of the two bases.

With the SUPPOSER: Each of the following statements is true or false. In each case state whether you think it is true or false and why you think so.

In a trapezoid:

-- The length of the median equals one half of the large base.
-- The median is parallel to the two bases.
-- The length of the median is equal to the average of the lengths of the two bases.
-- The length of the median is equal to twice the length of the altitude.

This kind of problem can introduce students to the fact that there are false statements and that gathering evidence and looking for counterexamples is essential before reaching conclusions.

Exploring Open-Ended Problems

In the textbook: A theorem: The median of a trapezoid is parallel to the bases and its length equals half the sum of the lengths of the bases.

An exercise: MN is the median of trapezoid ABCD, find the values of ....

With the SUPPOSER: List as many conjectures as you can about the segment which connects the midpoints of the two non-parallel sides in a trapezoid. Determine whether your conjectures hold true for all trapezoids or specific kinds of trapezoids. Prove at least one of your conjectures.

Do medians in other quadrilaterals have similar properties? Which properties hold for which types of quadrilaterals?

B. FOUR APPROACHES TO QUADRILATERALS

Here are four possible strategies for working with the SUPPOSER:

--Classifying Quadrilaterals by Diagonals
--Moving from Triangles to Quadrilaterals
--The Relationships between Quadrilaterals and Circles
--Making Your Own: What's Behind Some SUPPOSER Options?

Classifying Quadrilaterals by Diagonals

Most classification schemes for quadrilaterals are based on the properties of their sides and angles, paralleling the classification of triangles. However, it may be useful to work with a different scheme: classifying convex quadrilaterals by the properties of their diagonals.

In this scheme, there are five conditions:

1. The diagonals are equal in length.
2. The angle between the diagonals is a right angle.
3. The diagonals bisect each other.
4. Only one diagonal is bisected.
5. The diagonals intersect so that the ratios of the segments created by the intersection are equal.

Here is a chart of these conditions and the resulting shapes:

	CONDITIONS				
SHAPES	#1	#2	#3	#4	#5
Parallelogram			X		
Rectangle	X		X		
Rhombus		X	X		
Square	X	X	X		
Kite		X		X	
Trapezoid					X
Isosceles Trapezoid	X				X

This scheme is useful for two reasons. First, most students expect and/or seem to think that diagonals in a quadrilateral are equal and are bisectors (in fact this is the exception more than the rule). Second, an understanding of the properties of diagonals can simplify a number of proof-making and problem-solving tasks since it is often easy to demonstrate that diagonals bisect, but difficult to prove that angles and sides are equal.

Here are two ways to get students thinking about this before they start working with the SUPPOSER:

-- Have each student play with two straws, dowels, or coffee stirrers and let them make conjectures about the kinds of quadrilaterals that will be created by changing the relationships between the two sticks--the ratios of the lengths (where you cross one stick over the other) and the angles between the sticks. If you like, you can have students connect the four ends of the sticks with string in order to see more vividly the shapes that appear as they move the sticks around.

-- Ask students to devise rules for making a kite. The rules are: The angle between the diagonals is 90 degrees, one diagonal must be bisected, and the lengths are irrelevant.

Then, using the SUPPOSER, ask them to define the shapes created by two diagonals that bisect one another. Have the students collect their data in a table like the one below that is only a partial listing of all the possible cases. (If students are not familiar with the names of shapes, let them draw the shapes on their charts.)

Angle between Diagonals	Ratio of lengths of Diagonals	Shape
90	4:4	Square
60	4:4	Rectangle
40	5:5	Rectangle
90	4:7	Rhombus
60	4:7	Parallelogram
40	4:5	Parallelogram

In addition to learning about quadrilaterals, this activity provides an opportunity to talk about data collection, organization, and analysis when a problem has several variables. As posed, this problem has two variables (the angle between the diagonals and the ratio of the lengths of the diagonals). Two other variables are being held constant (ratios of the lengths created by the intersection of the diagonals). Having to deal with four variables would likely overwhelm students and demand a major portion of the semester.

<u>Moving from Triangles to Quadrilaterals</u>

Most geometry textbooks look at the triangles that exist within quadrilaterals. The SUPPOSER makes an alternative approach possible: taking the elements and theorems specific to triangles and exploring whether they can be generalized to quadrilaterals. As indicated above, students' grasp of the content of geometry is shape-specific: they may understand triangles, they may understand quadrilaterals, they may understand circles, but the connective tissue, the relationships among polygons and shapes, seems to elude them. Here is an opportunity to start making conjectures about "n"-gons and to begin to see which theorems hold true for all shapes and which are shape-specific. At a broader level, it is an occasion for students to develop inductive reasoning skills, and to learn how to move from the particular to the general and from the individual to the universal.

Here are some suggestions for pursuing this approach:

1. Midsegments: In a triangle, drawing the midsegments creates four equal triangles, each of which is similar to the original triangle. What happens when you draw the four midsegments of a quadrilateral?

   a. Start with parallelograms and using <u>Measure</u>, investigate the similarity of the two quadrilaterals and the relationship between the areas and the perimeters of the quadrilaterals. Working with the <u>Repeat</u> and <u>Measure</u> options, try to discover whether these relationships hold for all types of parallelograms. What about other classes of quadrilaterals?

      Students should discover that regardless of which quadrilateral they begin with, the new shape is a parallelogram, that its area is 1/2 the area of the original, and that the two quadrilaterals are not similar.

   b. To continue in this direction, construct yet another quadrilateral within the second shape, again using midsegments, as indicated in examples on pages 72 and 75.

      The relationship of the third quadrilateral to the original is as follows: its area is 1/4, its perimeter 1/2, and they are similar.

   c. How might these findings be thought of as generalizations of known theorems in the domain of triangles?

2. Angle Bisectors: In a triangle, the three angle bisectors intersect at a single point. What happens when you draw the four angle bisectors in a quadrilateral? (Angle bisectors are a more complicated matter in quadrilaterals than they are in triangles. Therefore, it is probably wise to offer more structure for the inquiry.)

Start by asking students to draw four angle bisectors in a quadrilateral, to repeat the construction on a variety of quadrilaterals, and to record their findings.
Some suggestions for things to look for: under what circumstances are the bisectors parallel, when do they intersect, when are two of them the same line?

After students collect some data, ask them to make conjectures based on their findings. For example, what are the properties of those quadrilaterals whose angle bisectors intersect in a common point? What are the properties of those whose bisectors create a square?

The Relationships between Quadrilaterals and Circles

Students are often surprised to learn that not all quadrilaterals can be circumscribed by a circle or contain an inscribed circle. The SUPPOSER enables students to gather evidence for making conjectures about circles and quadrilaterals with certain properties.

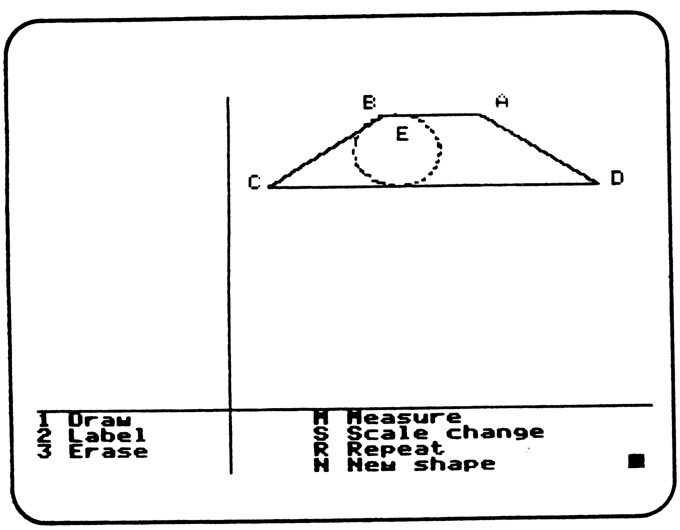
What are the properties of a quadrilateral in which it is possible to draw an inscribed circle?

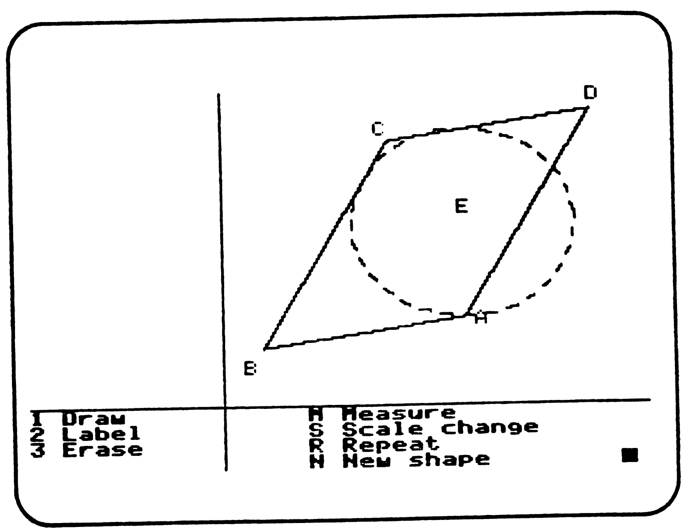
(A quadrilateral can be circumscribed by a circle if the sum of two opposite angles equals 180 degrees.
A quadrilateral can contain an inscribed circle if the sum of the lengths of two opposing sides equals the sum of the lengths of the other two opposing sides.)

Start the discussion by reviewing pictures of different quadrilaterals and asking students to guess which ones can contain an inscribed circle or be circumscribed by a circle. The likely conjecture will be that only quadrilaterals with four equal sides can contain a circle or be circumscribed by a circle. Using the Quads/Circles option under the New Shape menu, you can demonstrate that this is not the case. After showing that this conjecture is false, ask students to collect data and make conjectures about the relationships between quadrilaterals and circles.

Shape	Lengths AB	BC	CD	DA	Angles ABC	BCD	CDA	DAB	Circles Circ	Inscr	Other

Another approach is to have students look at quadrilaterals that cannot be circumscribed by a circle or contain an inscribed circle. Using the Circle option under Draw, the SUPPOSER will attempt to circumscribe or inscribe a circle in relationship to the quadrilateral--even if the circle can only pass through three vertices or is tangent only to three sides. Ask students to suggest how a quadrilateral would have to be changed in order to accommodate an inscribed or circumscribed circle. For example, in the figures below, students might conclude that if the bases of the trapezoid were smaller, or if the sides of the parallelogram were larger, these could contain inscribed circles. Let students work with some of these examples and try to construct quadrilaterals that can be circumscribed or contain an inscribed circle using the Sides and Angles option. Then using the Circle option under Draw, they can see whether their quadrilaterals can be circumscribed or can contain an inscribed circle. Ask them to make some conjectures based on the experience. Then, let them use the Quads/Circles option and take measurements to test their conjectures.





Making Your Own: What's Behind Some SUPPOSER Options?

Many of the constructions and options that the SUPPOSER will carry out with a few keystrokes can be constructed "manually," using more basic elements and options. This is a useful strategy because it requires that students grasp some fundamental concepts and theorems.

1. How can you inscribe a circle in a parallelogram without using the inscribed option under Circle on the Draw menu?

To solve this problem, students need to understand that:

- -- The center of an inscribed circle must be equidistant from all four sides.
- -- Every point on an angle bisector is equidistant from the rays of the angle.

Therefore, the point where the four angle bisectors intersect will be equidistant from all four sides and is the center of an inscribed circle.

By drawing the four angle bisectors and repeating this construction on different quadrilaterals, students should see that angle bisectors do not intersect in a single point in every quadrilateral and understand that not every quadrilateral can contain an inscribed circle.

2. Can you construct a kite using one or both of the options under Your Own/New Shape?

To construct a kite using the Diagonals method, students need to understand that in a kite the angle between the diagonals must be 90 degrees and that one diagonal must be bisected.

To construct a kite using the Sides and Angles method, students need to understand that in a kite, the two pairs of sides must be of equal length and one pair of the angles must be equal.

One way to approach this problem is to let students make constructions and measurements on kites, ask them to make conjectures about the properties of kites, and then ask them to construct their own kites. A similar approach can be taken with other types of quadrilaterals.

IV. EXAMPLES AND SUGGESTED EXERCISES

Most of the exercises and problems that follow involve making conjectures. Many students have difficulty making conjectures. Further, many students have difficulty with notions of plausibility and proof. Here are some suggestions for introducing these issues.

To begin with, it is suggested that you discuss the following propositions with your students.

- --If you believe a conjecture to be FALSE, you must offer a counterexample in order to disprove it.
- --In order to prove a conjecture FALSE, it is <u>sufficient</u> to offer a single counterexample.
- --If you believe a conjecture to be TRUE, you should be able to offer a convincing argument to support your belief.
- --In order to prove a conjecture TRUE, it is <u>not sufficient</u> to offer examples of cases that <u>are consistent</u> with the conjecture.
- --Why is it sufficient to offer a single counterexample in order to disprove a conjecture, and yet no number of confirming examples is sufficient to prove a conjecture?

The following section contains a series of activities for students. Some of them are quite simple and may be worked through in the space of part of a class period or as part of a regular homework assignment. Others are more elaborate and should be thought of as a sort of mini-research project. Depending on the available time and the level of your students, you may want to break up exercises with several parts into smaller pieces.

In any convex quadrilateral, the two diagonals create four triangles. What are the relationships among these four triangles?

Here is a chart you may find useful for organizing your findings. Just check off the columns which are true for each shape.

PROPERTIES OF TRIANGLES

TYPE OF QUAD	4 Congruent Triangles	2 Congruent Triangles	Similar Triangles	Equal Areas	Other
Square					
Rectangle					
Rhombus					
Parallelogram					
Kite					
Isosceles Trapezoid					
Trapezoid					

Draw the segments which connect the midpoints of the four sides of a parallelogram. State conjectures about the quadrilateral created by the four segments. What are the relationships between the two quadrilaterals? Consider the types of quadrilaterals, the perimeters, the sizes of the angles, the areas.

Repeat the process for other types of quadrilaterals.

Divide the sides of a square ABCD into three equal parts. Draw segments EG, GI, IK, and KE. Compare the two quadrilaterals and state your conjectures. Repeat the process for other quadrilaterals.

Suppose you divide the sides of ABCD into four equal parts, five equal parts, etc.

E is a random point inside quadrilateral ABCD. State a conjecture about the relationship between the perimeter of ABCD and the sum of the lengths of the segments connecting point E to the four vertices. Does your conjecture hold true if you substitute a triangle for a quadrilateral?

In parallelogram ABCD, bisect two adjacent angles (e.g., angles ABC and BCD). State conjectures about the relationship between the two bisectors and the point where they intersect.

Repeat the constructions on different types of parallelograms to test your conjectures.

For right, acute and obtuse triangles, you can state whether the sum of the squares of two sides is equal to, greater than, or less than the square of the third side.

For different quadrilaterals, can you identify a relationship between the sum of the squares of the sides and the sum of the squares of the diagonals?

Try starting with a square and a rectangle whose diagonals form right angles, and then move on to other quadrilaterals.

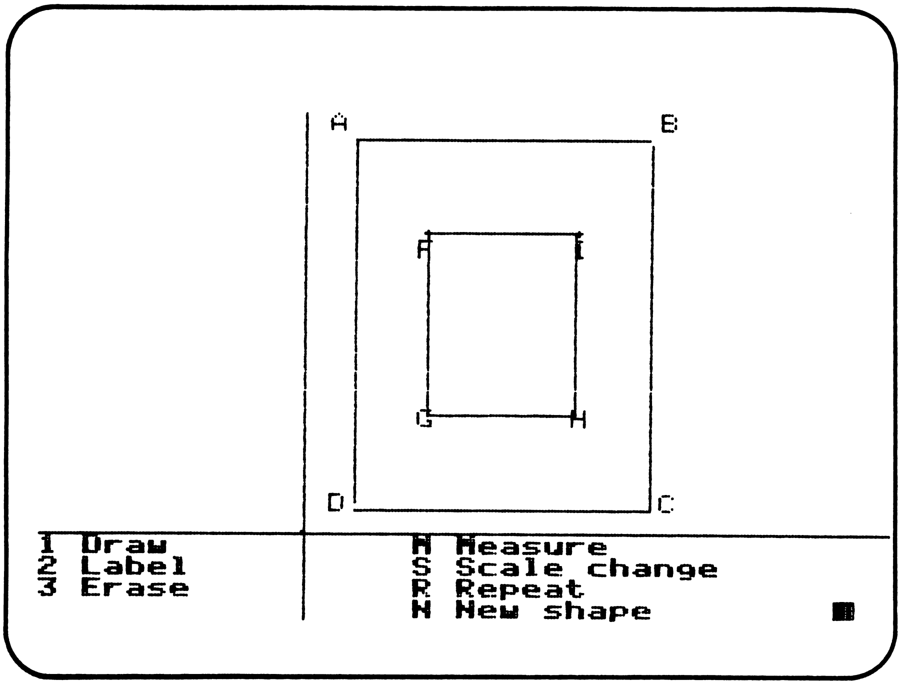
Define a median in a quadrilateral as the segment formed by connecting the midpoints of two opposite sides.

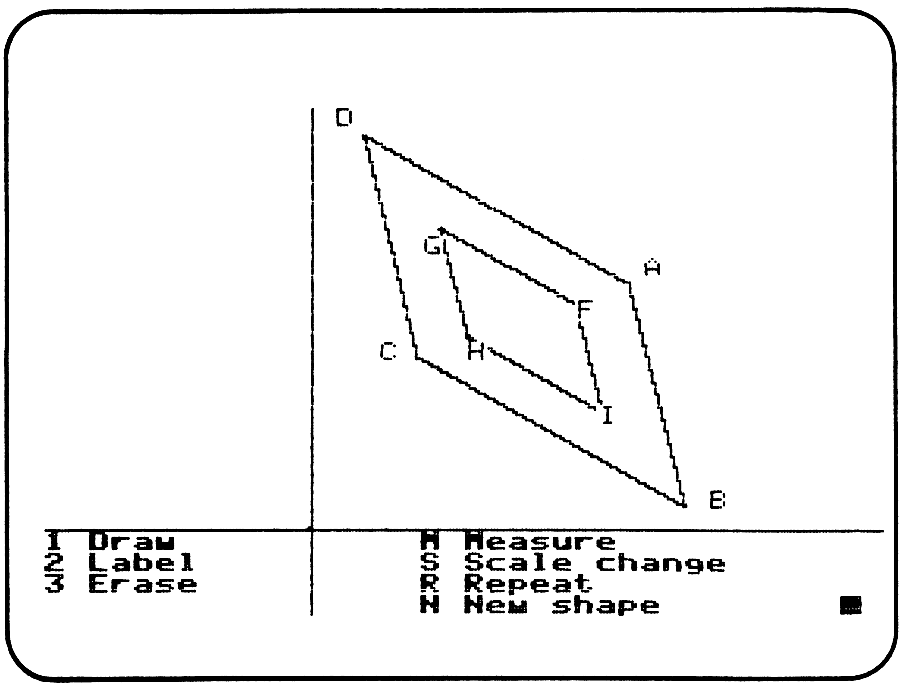
Two medians will always intersect. What are the special properties of the point where they intersect?

When the diagonals in a quadrilateral do not intersect at their midpoints, you can draw a segment connecting their two midpoints. What are the special properties of this segment?

A square, a rhombus, and a kite are quadrilaterals which share the following property: the angle between their diagonals is a right angle. Quadrilaterals with this property are called orthodiagonal quadrilaterals. Using the Diagonals/Your Own option, draw some additional orthodiagonal quadrilaterals and try to discover other special properties of this class of shapes.

Using the SUPPOSER, how can you draw the following figures?





In parallelogram ABCD, E is the midpoint of side CD. Segments AE and BE divide the parallelogram into three triangles. What are the relationships among triangles ADE, AEB, and BCE? State your conjectures.

Do these conjectures hold for other quadrilaterals? Prove one of your conjectures.

WORKING WITH THE APPLE II

(1) Turn on the television or monitor.

(2) Insert the diskette into the disk drive with the label facing up and on the right.

(3) Close the door to the disk drive.

(4) Turn on the Apple.

(5) You will see a red light on the disk drive turn on. If the disk drive light does not turn off in about 10 seconds, turn the Apple off and make sure your diskette is placed correctly in the drive.

(6) SUNBURST will appear on the screen, followed by the program name.

(7) Follow the directions in the program.

Shutting Off the System

(1) Remove the diskette from the disk drive and return it to its place of storage.

(2) Turn off the Apple.

(3) Turn off the television or monitor.

WHAT HAPPENS IF...?--SUNBURST COURSEWARE AND WARRANTY

1. What happens if a program will not load or run?
   Call us on our toll-free number and we will send you a new diskette.

2. What it I find an error in the program?
   We have thoroughly tested the programs that SUNBURST carries so we hope this does not happen. But if you find an error, please note what you did before the error occurred. Also, if a message appears on the screen, please write the message down. Then fill out the evaluation form or call us with the information. We will correct the error and send you a new diskette.

3. What happens if the courseware is accidentally destroyed?
   SUNBURST has a lifetime guarantee on its courseware. Send us the product that was damaged and we will send you a new one.

4. Can I copy this diskette?
   The material on the diskette is copyrighted. You <u>should</u> <u>not</u> copy the courseware.

5. Can I take this diskette out of the computer after the program has loaded and put it into another computer?
   Yes!