ESI MIND TOOLS ${ }^{\text {TM }}$ SERIES

FINANCIAL PLANNING FOR MULTIPLAN ${ }^{\text {TM }}$ AND THE APPLE II ${ }^{\text {® }}$

Expert Systems, Inc.

## FIRST EDITION <br> FIRST PRINTING—1982

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Edited by: Jim Rounds

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## Foreword

The electronic spreadsheet has become a standard working tool for managers and executives. "Financial Planning for Multiplan ${ }^{\mathrm{TM} * "}$ introduces you to the latest generation of electronic spreadsheet calculators and describes the 17 financial planning and forecasting programs, called calculators or worksheets, that are contained on the diskette accompanying the text. Each calculator is a complete and ready to use program that solves a family of related financial problems with Multiplan ${ }^{\mathrm{TM}}$ on your Apple $\mathrm{II}^{\oplus *}$ computer.

The text introduces and explains each calculator at a conceptual level and then illustrates the process of using it with a graphics walkthrough and a typical screen. For convenience of reference, all of the walkthroughs and screens are collected together in the Quick Reference Guide. Exercises are included for each calculator in the main body of the text. These exercises are designed to extend your knowledge of the concepts illustrated and demonstrate the interactive nature of each of the calculators.

Multiplan and the calculators used with it are tools that do much more than automate tasks that were previously done with paper, pencil, and a hand-held calculator. They are creative resources that free us from the drudgery of repetitive calculations and easily allow us to explore possibilities, to move rapidly from a general idea to a specific solution. They allow us a consistency of reporting and continuous updating that addresses the two most important factors in a manager's working environment: response time and productivity.

The cost of these new tools in terms of dollars and time spent learning to use them effectively is small in relation to their benefits. This has been an important part of their widespread acceptance. This collection of calculators is the outgrowth of five years experience teaching managers the basics of accounting, finance, and statistics. Most of these tools were first developed with $\mathrm{VisiCalc}^{\oplus}$ and now have been enhanced with the more versatile features of Multiplan. The calculators were selected for use by managers on the basis of their general validity for

[^0]solving practical financial problems in personal and business applications. Certainly one of the best ways to learn Multiplan ${ }^{\text {TM }}$ is to use well-designed calculators that are immediately applicable to a wide range of problems.

Jerry Jensen
Director of Education Expert Systems, Inc.

# FINANCIAL PLANNING FOR MULTIPLAN 

Software and System Requirements To Use This Product
SOFTWARE REQUIREMENTS
Microsoft Multiplan for the Apple II

1. Boot Diskette
2. System Diskette
3. Multiplan User's Manual

## SYSTEM REQUIREMENTS

Minimum Configuration for the Apple II

1. 64 K Memory
2. 16-Sector DOS
3. One Disk Drive
4. 80 -Column Card*

- Microsoft Multiplan Apple DOS Version

[^1]
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ANNUITY DUE CALCULATOR ANNDUE
CONTINUOUS ANNUITY CALCULATOR ..... CANN
AMORTIZATION SCHEDULE CALCULATOR ..... AMORT
Discounted Cash Flows
NET PRESENT VALUE CALCULATOR ..... NPV
INTERNAL RATE OF RETURN CALCULATOR ..... IRR
FINANCIAL MANAGEMENT RATE OF RETURN CALCULATOR ..... FMRR
Profit Planning ToolsBREAK-EVEN ANALYSIS CALCULATORBREAK
DEPRECIATION CALCULATOR ..... DEP
Statistics
STATISTICAL CALCULATOR ..... STAT
LINEAR REGRESSION CALCULATOR ..... LINREG
Real Estate Finance
VARIABLE RATE MORTGAGE CALCULATOR ..... VRM
GRADUATED PAYMENT MORTGAGE CALCULATOR ..... GPM
GRADUATED PAYMENT ADJUSTABLE MORTGAGE CALCULATOR ..... GPAM
WRAPAROUND MORTGAGE CALCULATOR ..... WRAP


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## Chapter 1

## Operating Procedures

### 1.1. HOW TO USE THIS TEXT



If you are a first time user of Multiplan, read through this chapter and follow the directions given in each section. Otherwise, study the flow diagram below in Fig. 1-1 to see how you are to proceed.


Fig. 1-1

### 1.2. HOW TO GET STARTED WITH MULTIPLAN

If this is the first time you have used Multiplan you should turn to the introductory chapter of your Multiplan User's Manual titled "Operating Information" and read "How to Start Multiplan the First Time."

Follow the directions there to make a copy of the Boot Diskette and at least one copy of the System Diskette.

In addition to these operations, it is necessary to convert your Financial Planning for Multiplan Diskette into a System Diskette consistent with your version of Multiplan. To do this, you must load a copy of the MP.SWAP file from one of your System Diskettes (the source disk) onto the Financial Planning for Multiplan Diskette (the destination disk). This may be done with the FID program on your DOS 3.3 SYSTEM MASTER diskette. Step-by-step instructions for this procedure are given in the walkthrough shown in Fig. 1-2.

It is also a good idea at this point to read the section on fundamentals in the first chapter, "Using Multiplan," in your Multiplan User's Manual. This will give you a feel for the structure of the Multiplan screen and how commands are executed from the keyboard.

### 1.3. HOW TO GET STARTED WITH FINANCIAL PLANNING

Your Financial Planning for Multiplan Diskette is a Multiplan System Diskette with 17 financial calculators stored on it. To access one of these calculators you must

1. Boot up with the Financial Planning for Multiplan Diskette.
2. Load a calculator on your worksheet, and
3. Set the manual recalculation mode.

Figs. 1-3, 1-4, and 1-5 walk you through these three operations step-by-step. Refer to these figures now and perform the operations described. You will then be ready to use the calculator that you loaded from your Financial Planning for Multiplan Diskette. For the purpose of this exercise we will assume you loaded COMP, the compound growth calculator. An alternative method of loading a calculator using the file directory is illustrated in Fig. 1-5.

### 1.4. USING THE FINANCIAL PLANNING CALCULATORS

You should now have COMP loaded and ready to use. The next step

## COPYING MP.SWAP TO THE FINANCIAL PLANNING DISKETTE

## INITIAL CONDITIONS:

(1) Power OFF on your Apple II computer.
(2) Power ON on your TV monitor.

## PROBLEM DEFINITION:

## ACTION ON KEYBOARD

## RESULT OR RESPONSE

 ON SCREEN

Fig. 1-2

## BOOTING UP WITH THE FINANCIAL PLANNING FOR MULTIPLAN DISKETTE

NOTE: The first time you start Multiplan, follow the steps in your Multiplan manual "How to Start Multiplan the First Time".

INITIAL CONDITIONS:
(1) Power OFF on your Apple II
(2) Power ON on your TV monitor

PROBLEM DEFINITION: Start up Multiplan with the BOOT DISKETTE and the FINANCIAL PLANNING FOR MULTIPLAN diskette. The screen will display a blank Multiplan worksheet.

## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN



Fig. 1-3

## LOADING A CALCULATOR ON YOUR WORKSHEET

## INITIAL CONDITIONS: <br> Multiplan booted up with the FINANCIAL PLANNING FOR MULTIPLAN diskette.

PROBLEM DEFINITION: Load a calculator from the FINANCIAL PLANNING FOR MULTIPLAN diskette onto the Multiplan worksheet. The compound growth calculator, COMP, will be loaded in this example. There are 16 other calculators to choose from on this diskette.


Fig. 1-4
is to learn how to use the COMP calculator. Each of the 17 financial calculators is explained in Chapters Three through Eight. Each chapter contains a general introduction to the concept behind each calculator, a flow diagram to walk you through a typical problem, and examples for you to work that will further illustrate principles and concepts.

You should now turn to Section 3.2 in Chapter Three, "How To Use COMP," that walks you through a typical application with COMP. Go through this example now and then try a few variations of your

## SETTING THE MANUAL RECALCULATION MODE

## INITIAL CONDITIONS:

(1) Multiplan booted up
(2) A calculator is loaded and ready to use

PROBLEM DEFINITION: Switch from the automatic calculation mode that is on when Multiplan is booted up to the manual recalculation mode.


Fig. 1-5
own. All of the calculators are easy to use once you have grasped the basic concept of how they are constructed; the flow diagrams which walk you through an application of each calculator were designed to convey this idea as quickly and easily as possible. They are all collected for your convenience in the "Quick Reference Guide."

## LOADING A CALCULATOR FROM THE FILE DIRECTORY

INITIAL CONDITIONS:
Multiplan booted up with the FINANCIAL PLANNING FOR MULTIPLAN diskette.
PROBLEM DEFINITION: Display the file directory on the screen and load a calculator.

## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN

| Press: T, L, and Ctrl-D | Directory appears on <br> screen. |
| :--- | :--- |
| Using the direction keys*, <br> move the cursor to the cell <br> with the name of the <br> calculator you wish to load. Enters the filename indicated <br> and executes the transfer <br> command to load the file from <br> the diskette into the computer. <br> Press: RETURN  |  |

*See your Multiplan User's Manual for an explanation of how to use the direction keys.

Fig. 1-6

## Chapter 2

## Creating Your Own Calculators

### 2.1. CREATING YOUR OWN CALCULATORS

I. PLANNING PHASE

PROBLEM DEFINITION
A. State Problem Conceptually
B. Define Input and Outputs

## II. DESIGN PHASE

SCREEN DESIGN
A. Define What Information It Must Contain

1. Input and Output Data
2. Cell Definitions, Titles, or Headings
3. Prompt and Direction for Users
B. Format Screen for Ease of Use

## ALGORITHM SELECTION

Select Algorithms To Process
Numerical and Textual Data
III. PRODUCTION PHASE PRODUCTION
A. Construct Calculator
B. Test for Completeness
C. Document Calculator


The 17 Financial Planning calculators are general purpose and ready to use; at some point, however, you may wish to modify some of them for your own special needs, or build similar calculators that perform completely different functions. Chapter 2 was designed to help you develop a systematic approach to creating your own calculators.

An electronic spreadsheet allows you to design your own worksheets on the screen and save them as files on a floppy disk. We call these files calculators, or worksheets, because they define a definite structure within which to work and perform your calculations. Your worksheets will be exercises in exploring an idea, or learning some new aspect of the spreadsheet, or it will attempt to solve a specific problem by automating a routine task which can be done more efficiently and, usually, more accurately with a computer. Your attitude and your mental set when you approach this task can add immeasurably to your effectiveness and your own personal satisfaction in solving problems and in using your computer.

If you look on creating calculators as a challenge that will save you time and engage your creative faculties in problem solving, it will never become a routine task. In a small way you can enjoy all of the tension and satisfaction of discovery that any creative activity involves. Many common elements in any problem-solving situation can be isolated and used in carrying out a project. We can define a methodology that will guide us in formulating our ideas and executing our plan without limiting our scope in any way. This is the purpose of this chapter, to outline a method of problem solving which will put discipline into your efforts at producing your own calculators.

We may identify three phases in creating a calculator: a planning phase, a design phase, and a production phase.
PLANNING DESIGN PRODUCTION

It is important, in creating calculators, to make a clear distinction between technical and conceptual levels of expression. We build calculators in order to solve a specific problem or a family of related problems. The first step is to define our problem. This is the planning phase and should be done at a conceptual, not a technical level. This does two things:

1. It focuses our attention on the overall flow and function of our program, and
2. It prevents us from getting caught up in technical details as we approach our problem.
An essential part of our problem definition is an exact specification of
the inputs, the numerical and textual data that must be entered into the computer to solve the problem, and the outputs, the data that constitutes a solution to the problem. We will see many specific examples of this in later chapters.

We may summarize the planning phase in the following table.
PROBLEM DEFINITION
A. State problem conceptually
B. Define inputs and outputs

The design phase accomplishes two things: it defines exactly what we see on the screen and how that information is to be processed to produce a solution to our problem. Obviously, screen designs must include the input and output data, but they must also include titles and headings and possibly directions for users. Of equal importance here is the organization of the screen itself. It should be formatted for ease of use with a logical and visually pleasing presentation. Overcrowded and poorly organized screens are self-defeating, they frustrate and intimidate end users. Even well-designed calculators usually require some user training, and it is helpful to include prompts on the screen to coach the user on what to do next.

## SCREEN DESIGN

A. Define what information it must contain.

1. Input and output data.
2. Cell definitions, titles, or headings.
3. Prompts and directions for users.
B. Format screen for ease of use.

The next step of the design phase is to determine how we are to process the input data in order to produce the desired output data. Any procedure that does this is called an algorithm. We must therefore select appropriate algorithms to process the numerical and textual inputs of our program.

## ALGORITHM SELECTION

Select algorithms to process numerical and textual data.

The final phase is the production of the calculator. Our objective here is to produce a program that is easy to use, does what it is supposed to, and has some supporting documentation to instruct new users how to use it and, if necessary, how to modify it. This last step requires some description of the structure of the calculator in terms of its organization and content.

An important step in the production phase is testing the program to verify that it is doing what it was designed to do. This is called debugging the program. The best way to prevent bugs in your program is to do some advance planning. You should have some manual results already computed that you know are correct to use as a standard against which to test the program. In addition to this, invent some extreme cases for data input to see if the calculator is as general as you think it is. If possible you want it to be smart enough to detect errors when bad data is input. This is called error trapping. For instance, if you have built a table for the program to read, include an error message wherever possible to flag unacceptable data.

## PRODUCTION

A. Construct calculator
B. Test for completeness
C. Document calculator

The preceding description is somewhat clinical in its discussion of what is essentially a very creative and dynamic process. These three stages in creating a calculator are really not linearly dependent in the sense conveyed. They are functionally related and interdependent. You may realize, while designing your screens, that the problem definition was incomplete or, while selecting algorithms, you may discover that you have to change the screen design to accommodate certain computations.

The message here is that when you have finished you will have gone through a process that includes all of these phases. It is a cyclical process and in all likelihood you will go through these phases several times as you refine your concept. Experienced programmers will often sit down at their computer and hammer out a basic program that will
clarify for them what needs to be done. They will then scrap this program and systematically develop the final product. This is one approach. Find the approach that works best for you and use the outline provided here to structure your work so that you will be productive and successful in creating your own calculators. A summary of this outline may be found at the beginning of this chapter.

## Chapter 3

## Compound Growth

### 3.1. COMP: A COMPOUND GROWTH CALCULATOR



The purpose of this calculator is to solve the compound growth problem. In its simplest form, this problem arises in the following way. Suppose you deposit $\$ 1,000$ in savings for a period of one year. If you earn $12 \%$ interest per year on your savings, you will earn $\$ 120$ in one year. This is simple interest. Suppose, instead, that you earn $1 \%$ per month on your savings and this interest is compounded. In the first month, you will earn $1 \%$ of $\$ 1,000$ or $\$ 10.00$. You then have $\$ 1,010$ and, in the second month, you will earn $1 \%$ of $\$ 1,010$, or $\$ 10.10$. Table 3-1 shows an annual summary of compound interest.

| PERIOD | SAVINGS AT <br> BEG OF PER | INTEREST <br> EARNED | ACCUMULATED <br> INTEREST |
| ---: | ---: | ---: | ---: |
| 1 | $\$ 1000.00$ | $\$ 10.00$ | $\$ 10.00$ |
| 2 | $\$ 1010.00$ | $\$ 10.10$ | $\$ 20.10$ |
| 3 | $\$ 1020.10$ | $\$ 10.20$ | $\$ 30.30$ |
| 4 | $\$ 1030.30$ | $\$ 10.30$ | $\$ 40.60$ |
| 5 | $\$ 1040.60$ | $\$ 10.41$ | $\$ 51.01$ |
| 6 | $\$ 1051.01$ | $\$ 10.51$ | $\$ 61.52$ |
| 7 | $\$ 1061.52$ | $\$ 10.62$ | $\$ 72.14$ |
| 8 | $\$ 1072.14$ | $\$ 10.72$ | $\$ 82.86$ |
| 9 | $\$ 1082.86$ | $\$ 10.83$ | $\$ 93.69$ |
| 10 | $\$ 1093.69$ | $\$ 10.94$ | $\$ 104.62$ |
| 11 | $\$ 1104.62$ | $\$ 11.05$ | $\$ 115.67$ |
| 12 | $\$ 115.67$ | $\$ 11.16$ | $\$ 126.83$ |

## Table 3-1

With compound growth you can earn interest on the amount deposited plus accumulated interest to the current period. At the end of a year the net effect is that you earned $\$ 126.83$, compounding monthly at $1 \%$ as opposed to $\$ 120.00$ you would have earned with simple interest at $12 \%$ per annum. In both cases, the nominal interest rate is $12 \%$. The effective yield is greater with compounding and is
equivalent to earning $12.683 \%$ simple interest per year. Thus, the more frequent the compounding, the greater the effective yield.

Traditionally, the compound growth formula is expressed in the form:

$$
\mathrm{FV}=\mathrm{PV}(1+\mathrm{P})^{\mathrm{N}}
$$

Where,
PV is the present value ( $\$ 1000$ in our example),
FV is the future value ( $\$ 1126.83$ ) after N periods ( 12 months) compounded at a periodic interest rate $P$ per period ( $1 \%$ ), $P$ is expressed as a decimal (0.01) in this equation.

The compound growth equation expresses the relationship between four variables: PV, FV, P, and N. Normally we are not given F, the periodic interest rate. It is customary to give the annual interest rate or nominal interest rate I and the frequency of compounding F ; and then compute P with the formula:

$$
P=\frac{I}{F}
$$

For example, if $\mathrm{I}=12 \%$ and we are compounding monthly, then $\mathrm{F}=$ 12 and $P=12 \% / 12=1 \%$. If we compound quarterly; then $F=4$ and $P=12 \% / 4=3 \%$.

In the most general case we are interested in the relationship between the five variables: F, N, I, PV, and FV. The compound growth problem may thus be expressed symbolically as a functional relationship between five variables:

$$
\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})=0
$$

We are not interested, at this point, in the technical details of solving such a symbolic equation; we wish only to express conceptually the nature of the problem. In some cases, we may not be able to solve the problem because of technical limitations, but it is usually valuable to state the problem in the most general case.

Given this relationship we may define symbolically the five possible situations which arise in compound growth problems.

## 1. FREQUENCY OF COMPOUNDING

$$
\mathrm{F}=\mathrm{f}(\mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})
$$

2. TIME TO MATURITY

$$
\mathrm{N}=\mathrm{f}(\mathrm{~F}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})
$$

3. ANNUAL INTEREST OR GROWTH RATE

$$
\mathrm{I}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{PV}, \mathrm{FV})
$$

4. DISCOUNTING A FUTURE VALUE TO A PRESENT VALUE

$$
\mathrm{PV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{FV})
$$

## 5. COMPOUNDING A PRESENT VALUE TO A FUTURE VALUE

$$
\mathrm{FV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PV})
$$

In each of these five cases, the variable on the left side of the equation is the output or computed value, given an input consisting of the four variables on the right-hand side of the equation.

We now have an analysis that allows us to design the screen on which our data are input and the resulting computations (output) are displayed. Many ways exist in which this can be done, but it is best to keep it logically simple and as graphic as possible. The particular solution presented here emulates the format you are familiar with in using hand-held calculators. Table 3-2 shows a typical screen.

Notice the problem entered on this screen is the same problem considered in Table 3-1. Also notice that the present value is negative. We will follow the standard accounting practice of entering a value as negative if it is a cash outflow and as positive if it is a cash inflow. In this case, we made a deposit of $-\$ 1,000$ and received $\$ 1,126.83$ at the end of one year. In order to incorporate this convention in our compound growth calculator, we must incorporate a sign convention in the compound growth formula. Therefore our formula becomes:

$$
0=\mathrm{PV}+\mathrm{FV} /(1+\mathrm{I} / \mathrm{F})^{\mathrm{N}}
$$

We have substituted " $\mathrm{I} / \mathrm{F}$ " for " P " in the original equation. This formula requires either the present value or future value be negative in order for the sum to be 0 . Solving this equation for each of the variables it contains gives us the following equations.

1. FREQUENCY OF COMPOUNDING

$$
\mathrm{F}=\mathrm{f}(\mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})=\mathrm{I} /(-\mathrm{FV} / \mathrm{PV})^{1 / \mathrm{N}}-1
$$

2. TIME TO MATURITY

$$
\mathrm{N}=\mathrm{f}(\mathrm{~F}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})=\mathrm{LN}(-\mathrm{FV} / \mathrm{PV}) / \mathrm{LN}(1+\mathrm{I} / \mathrm{F})
$$

3. ANNUAL INTEREST OR GROWTH RATE

$$
\mathrm{I}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{PV}, \mathrm{FV})=\mathrm{F}\left[(-\mathrm{FV} / \mathrm{PV})^{1 / \mathrm{N}}-1\right]
$$

4. DISCOUNTING A FUTURE VALUE TO A PRESENT VALUE

$$
\mathrm{PV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{FV})=-\mathrm{FV} /(1+\mathrm{I} / \mathrm{F})^{\mathrm{N}}
$$



## Table 3-2

### 3.2. HOW TO USE COMP: A COMPOUND GROWTH CALCULATOR

The basic concept in using all of the compound growth and annuity calculators is very simple. First, the given data is entered in the input column, then the variable that is to be calculated is entered in the
remaining input cell. The manual recalculation command is invoked, and the results, then, are computed and posted to the specified table. Follow the step-by-step procedure in Fig. 3-1. The corresponding screen is displayed in Table 3-3.


## Table 3-3

## Helpful Hints on Using COMP

A. On each of the compound growth and annuity calculators, a pointer< is used to flag the calculated value in the input column. If an error message \#VALUE appears in some of the output tables, you may have attempted to compute more than one value in the input column and there will be more than one pointer displayed there. For accurate results, you must enter only one variable in the input column and there will be only one pointer in the input column to reflect this condition.
B. Standard accounting practices regarding the sign conventions of cash flows are observed on all of the compound growth and annuity calculators. If you obtain unreasonable answers, you may have

## COMP: A COMPOUND GROWTH CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

## INITIAL CONDITIONS:

(1) COMP loaded and ready to use
(2) Manual recalculation set
(3) Cursor at POST TO TABLE cell, R2C9

PROBLEM DEFINITION: Compute the future value and post results to Table 2 with the following input.

$$
\begin{aligned}
\mathrm{F} & =12 \\
\mathrm{~N} & =120 \\
\mathrm{I} & =16 \% \\
\mathrm{PV} & =-1000
\end{aligned}
$$

## RESULT OR RESPONSE

ACTION ON KEYBOARD ON SCREEN

Enters ' 2 ' in the POST TO TABLE cell and moves cursor to the next input cell.

Enters '12' in the NUMBER OF PERIODS PER.YEAR cell and moves the cursor to the next input cell.

Enters ' 120 ' in the NUMBER OF PERIODS cell and moves the cursor to the next input cell.

Enters '16\%' in the ANNUAL INTEREST RATE cell and moves the cursor to the next input cell.

Enters '-1000' in the PRESENT VALUE cell and moves the cursor to the next input cell.

Fig. 3-1

## RESULT OR RESPONSE ON SCREEN

## ACTION ON KEYBOARD

> Enters the variable 'FV' in the FUTURE VALUE cell as a formula with the value command. There will be no value change until the sheet is recalculated.

```
Recalculates the
spreadsheet. Displays
'$4,900.94' in the FUTURE
VALUE cell and posts
results to Table 2.
```

Fig. 3-1 - cont.
entered a present value, a payment, or a future value with a sign inconsistent with your intentions. Assume the position of either the creditor or debtor, and then consistently give the correct sign to each cash value. Outflows are negative and inflows are positive.
C. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.

### 3.3. EXAMPLES USING COMP

1. Two sums of money can be compared in value only if we compute their values on the same date, relative to a specified interest or discount rate. Thus, we may ask: What is worth more at $9 \%, \$ 1,000$ today or $\$ 2,000$ in 8 years? Try this with annual and monthly compounding. (Hint: Let the $\$ 1,000$ be a PV, compute its FV in 8 years and compare this value with the $\$ 2,000$. Or, alternatively, find the present value of the $\$ 2,000$ and compare it with the $\$ 1,000$.)
2. If we are promised a future sum, we may ask what it is worth today, or what we would be willing to sell this promissory note for today. For instance, if you were promised $\$ 10,000$ in 30 years, and you think $10 \%$ is a reasonable discount rate, what is the present value of that future sum?
3. If you deposit $\$ 1,000$ now at $16.25 \%$ interest compounded quarterly (annually, monthly), what will it be worth in 15 years?
4. Your $\$ 80,000$ house is appreciating at about $12 \%$ per year. What will it be worth in 5 years? 10 years? 20 years?
5. At a rate of $14 \%$, how long will it take your investment of $\$ 3,500$ to grow to $\$ 10,000$ ?
6. At $18 \%$, how long will it take a sum to double? Triple?
7. You want to double your money in five years. What growth rate does this require? How about three years? Seven years?
8. You bought stock in 1972 at $\$ 20.00$ a share and in 1982 it is worth $\$ 56.00$ a share. What has been the average annual growth rate of your investment?

### 3.4. CCOMP: A CONTINUOUS COMPOUND GROWTH CALCULATOR

The purpose of this calculator is to solve the continuous compound growth problem. Suppose you deposit $\$ 1,000$ in savings for a period of one year. If you earn $12 \%$ interest per year on your savings, you will earn $\$ 120$ in one year. This is simple interest. As you may verify with COMP, if you earn $12 \%$ interest compounded semi-annually on $\$ 1,000$, your interest will be $\$ 123.60$. If you compound quarterly, your interest will be $\$ 125.51$. If you continue to compound interest at increasingly smaller intervals you might generate a table something like this:

## INTEREST EARNED ON \$1,000 IN ONE YEAR

| COMPOUNDING | INTEREST | INCREMENT |
| :---: | :---: | :---: |
| FREQUENCY | EARNED |  |
| 1 | . $\$ 120.00$ |  |
| 2 | . \$123.60 | . 3.60 |
| 4 | . $\$ 125.51$ | . 1.91 |
| 12 | . $\$ 126.83$ | .1.32 |
| 52 | . $\$ 127.34$ | . 0.51 |
| 365 | . $\$ 127.47$ | . 0.12 |
| 8760 | . $\$ 127.50$ | . 0.03 |
| 525, 600 | . $\$ 127.50$ | . 0.00 |

Although the compounding frequency increases by larger steps in each line of the preceding table, the increase in dollar earnings decreases steadily. In fact, when you move from compounding hourly to compounding by the minute, there is no difference in the dollar amount when rounding to the nearest cent. There is apparently an upper limit to what we can do to increase our interest earnings by compounding more frequently. This upper limit is called continuous compounding.

Compounding $\$ 1,000$ continuously for one year at $12 \%$ yields a future sum of $\$ 1,127.50$. In this particular case, there is no difference between continuous compounding and hourly compounding, and only $\$ 0.03$ difference occurs between continuous and daily compounding.

In practice, continuous compounding has the advantage of being very easy to use and, mathematically, is very easy to compute. For these reasons alone, it recommends itself as a standard model for financial growth problems. Conceptually it is the same model used widely in physics, chemistry, and biology to measure continuous growth.

The continuous growth equation may be stated in this form:

$$
\mathrm{FV}=\mathrm{PV} \exp (\mathrm{IT})
$$

Where,
PV is the present value ( $\$ 1,000$ in our example),
FV is the future value ( $\$ 1127.50$ ) after T years (one year) compounded continuously at a rate I ( $12 \%$ ).
I is expressed as a decimal (.12) in this equation.
In business it is convenient to express time in units other than years, such as months, quarters, etc. For this reason, we will express time in terms of periods ( N ) and the number of periods per year ( F for frequency). We then have the equation

$$
\mathrm{T}=\mathrm{N} / \mathrm{F}
$$

for time and we express the continuous growth equation in the form

$$
\mathrm{FV}=\mathrm{PV} \exp (\mathrm{IN} / \mathrm{F})
$$

One more convention must be added before we arrive at the desired relationship. We will follow the accounting practice of viewing cash outflows as negative, and cash inflows as positive, with regard to a particular transaction. In order for this to work, the two sides of the above equation must sum to 0 :

$$
0=\mathrm{PV}+\mathrm{FV} / \exp (\mathrm{IN} / \mathrm{F}) .
$$

Rewriting the equation in this form does not change the absolute values of any of the variables; it simply requires that the present value and future value always have opposite signs.

The continuous growth equation expresses the relationship between five variables: F, N, I, PV, and FV. The continuous growth problem may thus be expressed symbolically as a functional relationship between these variables:

$$
\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})=0 .
$$

Solving for these variables gives us these five situations.

1. FREQUENCY

$$
\mathrm{F}=\mathrm{f}(\mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})=\mathrm{IN} / \ln (-\mathrm{FV} / \mathrm{PV})
$$

2. TIME TO MATURITTY

$$
\mathrm{N}=\mathrm{f}(\mathrm{~F}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})=\ln (-\mathrm{FV} / \mathrm{PV}) \mathrm{F} / \mathrm{I}
$$

3. ANNUAL INTEREST OR GROWTH RATE

$$
\mathrm{I}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{PV}, \mathrm{FV})=\ln (-\mathrm{FV} / \mathrm{PV}) \mathrm{F} / \mathrm{N}
$$

4. DISCOUNTING A FUTURE VALUE TO A PRESENT VALUE

$$
\mathrm{PV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{FV})=-\mathrm{FV} / \exp (\mathrm{IN} / \mathrm{F})
$$

5. COMPOUNDING A PRESENT VALUE TO A FUTURE VALUE

$$
\mathrm{FV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PV})=\mathrm{PV} \exp (\mathrm{IN} / \mathrm{F})
$$

In each of the previous five cases, the variable on the left side of the equation is the output or computed value, given an input comprising the four variables on the right-hand side of the equation.

We now have an analysis that allows us to design the screen on which our data are input and resulting computations are displayed. Many ways exist in which this can be done. It is best to keep it logically simple and as graphic as possible. Table 3-4 shows a typical screen.

### 3.5. HOW TO USE CCOMP: A CONTINUOUS COMPOUND GROWTH CALCULATOR

The basic concept in using all of the compound growth and annuity calculators is very simple. First, the given data is entered in the input column, then the single variable that is to be calculated is entered in the remaining input cell. The manual recalculation command is invoked, and the results are computed and posted to the specified table. Follow the step-by-step procedure shown in Fig. 3-2. The corresponding screen is displayed in Table 3-4.

## Helpful Hints on Using CCOMP

A. On each of the compound growth and annuity calculators, a pointer < is used to flag the calculated value in the input column. If an error message \#VALUE appears in some of the output tables, you may have attempted to compute more than one value in the input column and there will be more than one pointer displayed there. For accurate

## CCOMP: A CONTINUOUS COMPOUND GROWTH CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) CCOMP loaded and ready to use
(2) Manual recalculation set
(3) Cursor at POST TO TABLE cell, R2C9

PROBLEM DEFINITION: Compute the future value and post results to Table 2 with the following input.

$$
\begin{aligned}
\mathrm{F} & =1 \\
\mathrm{~N} & =10 \\
\mathrm{I} & =16 \% \\
\mathrm{PV} & =-1000
\end{aligned}
$$

## RESULT OR RESPONSE ON SCREEN

ACTION ON KEYBOARD


Fig. 3-2

## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN



Fig. 3-2 - cont.


Table 3-4
results, you must enter only one variable in the input column and there will be only one pointer in the input column to reflect this condition.
B. Standard accounting practices regarding the sign conventions of cash flows are observed on all of the compound growth and annuity calculators. If you obtain unreasonable answers, you may have entered a present value, a payment, or a future value with a sign inconsistent with your intentions. Assume the position of either the creditor or debtor, and then consistently give the correct sign to each cash value. Outflows are negative and inflows are positive.
C. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.

### 3.6. EXAMPLES USING CCOMP

1. Two sums of money can be compared in value only if we compute their values on the same date, relative to a specified interest or discount rate. Thus, we may ask: What is worth more at $9 \%, \$ 1,000$ today or $\$ 2,000$ in 8 years with continuous compounding? (Hint: Let the $\$ 1,000$ be a PV, compute its FV in 8 years and compare this value with the $\$ 2,000$. Or, alternatively, find the present value of the $\$ 2,000$ and compare it with the $\$ 1,000$.)
2. If we are promised a future sum, we may ask what it is worth today, or what we would be willing to sell this promissory note for today. For instance, if you were promised $\$ 10,000$ in 10 years, and you discount it at the average inflation rate for that period, which you estimate to be $12 \%$, what is the present value with continuous discounting?
3. If you deposit $\$ 1,000$ now at $16.25 \%$, compounded continuously, what will it be worth in 5 years? 10 years? 15 years?
4. At a rate of $14 \%$, how long will it take your investment of $\$ 3,500$ to grow to $\$ 10,000$ with continuous compounding?
5. At $18 \%$ continuous compounding, how long will it take a sum to double? Triple?
6. You want to double your money in 5 years. What growth rate does this require? How about 3 years? 7 years?
7. You bought stock in 1972 at $\$ 20.00$ a share, and in 1982 it is worth $\$ 56.00$ a share. What has been the continuous compound growth rate of your investment?

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## Chapter 4

## Discounted Cash Flow Analysis: Annuities

### 4.1. INTRODUCTION TO CASH FLOWS



A cash flow is a schedule of cash transactions over a fixed period of time. Each transaction is either a cash inflow or a cash outflow. Inflows are positive (+), and outflows are negative ( - ). Assuming N payments, the sum of all the payments (PMT) of a cash flow is called its cash value (CV):

$$
\mathrm{CV}=\mathrm{PMT}_{1}+\mathrm{PMT}_{2}+\mathrm{PMT}_{3}+\ldots+\mathrm{PMT}_{\mathrm{N}}
$$

Since a cash flow is defined over a fixed period of time, and a date is associated with each payment, we may discount each payment to the beginning of the cash flow at a given discount rate. The sum of all such values is called the present value (PV) of the cash flow at that discount rate. If $P$ is the periodic interest rate,
$\mathrm{PV}=\mathrm{PMT}_{1} /(1+\mathrm{P})^{1}+\mathrm{PMT}_{2} /(1+\mathrm{P})^{2}+\ldots+\mathrm{PMT}_{\mathrm{N}} /(1+\mathrm{P})^{\mathrm{N}}$.
Similarly, we may compound the value of each payment to the end of the cash flow at a given interest rate. The sum of all such values is called the future value of the cash flow at that interest rate. If $P$ is the periodic interest rate,

$$
\mathrm{FV}=\mathrm{PMT}_{1}(1+\mathrm{P})^{\mathrm{N}-1}+\mathrm{PMT}_{2}(1+\mathrm{P})^{\mathrm{N}-2}+\ldots+\mathrm{PMT}_{\mathrm{N}} .
$$

If the interest rate is positive and the payment stream is taken as positive, the following relationship holds for any cash flow

$$
\mathrm{PV}<\mathrm{CV}<\mathrm{FV} .
$$

This mathematical statement says the debt that a stream of payments pays off is less than the sum of the payment stream and, in addition, both these sums are less than the amount that would accrue if the payment stream is regarded as earning interest at the given rate.

If we regard the cash flow as paying off a present debt, the accumulated interest paid over the life of the cash flow is:

## ACCUMULATED INTEREST $=\mathrm{CV}-\mathrm{PV}$.

If we regard the cash flow as accumulating to a future sum, then the interest earned over the life of the cash flow is:
EARNED INTEREST = FV - CV.

We may apply these same ideas to compute the accumulated interest or the earned interest to any payment in a cash flow.

Cash flows may be classified broadly as uneven cash flows or even cash flows. An even or level cash flow is a cash flow in which all the payments are the same. This is not the case in an uneven cash flow. The next chapter deals with uneven cash flows. In this chapter, we will discuss a special case of even cash flows called annuities.

An annuity is a cash flow that is even and periodic, i.e., all payments are of the same amount and are made at regular intervals. A mortgage loan for $\$ 80,000$ with payments of $\$ 875$ per month for 360 months is one of the most common examples of an annuity. By definition, an annuity establishes a uniform sequence of time intervals (periods) over a fixed interval of time. We may use this fact to classify annuities. We will consider three possibilities:

1. ORDINARY ANNUITY: payments are made at the end of each period,
2. ANNUITY DUE: payments are made at the beginning of each period, or
3. CONTINUOUS ANNUITY: a single payment amount is accumulated continuously throughout each period.

Ordinary annuities are normally used to pay a present debt with future payments. This is the process of amortization in which interest is paid only on the unpaid balance of the debt. Interest is accrued on the unpaid balance from the beginning of each period and is paid at the end of the period. Any amount of the payment over the interest directly reduces the principal.

Sinking funds are the most common example of annuities due. Here, regular level payments are made to accumulate a future sum, usually
for the purpose of retiring an anticipated debt. Interest is earned from the beginning of each period when the periodic deposit is made.

The classic example of a continuous annuity is a penny arcade that is open 24 hours a day and takes in an average of $\$ 5,000$ a month at a, more-or-less, continuous rate.

We will discuss each of these concepts, in turn, in the following sections. The discussion that follows applies equally to all three types of annuities.

## Functional Relationships

An annuity is a stream of N payments each of the amount PMT:

$$
\mathrm{PMT}_{1}, \mathrm{PMT}_{2}, \mathrm{PMT}_{3}, \ldots, \mathrm{PMT}_{\mathrm{N}} .
$$

Normally, when discounting or compounding an annuity, the compounding periods correspond with the dates that the payments are made. We will follow this practice in constructing our calculators and assume that F payments are made per year with a corresponding compounding frequency of F . Thus, if we associate a discount rate of $12 \%$ with the annuity and are making monthly payments, $\mathrm{F}=12$ and the periodic interest rate is $1 \%$. We will call the annual interest or discount rate associated with an annuity I. When discounting an annuity we compute its present value, PV, and when compounding an annuity we compute its future value, FV. This gives us six variables and the following functional relationship:

$$
\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{PMT}, \mathrm{FV})=0 .
$$

We will assume the usual accounting conventions with regard to cash flows so that outflows will be negative and inflows, positive. Strictly speaking, all six variables in our relationship are not needed because, when computing the present value of an annuity, the future value does not enter into the calculation:

$$
\mathrm{PV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PMT}) .
$$

Conversely, if we are computing the future value, the present value is not included:

$$
\mathrm{FV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PMT}) .
$$

Using all six variables, however, gives some definite advantages. In the first case, when computing the present value of an annuity, we can simply set $\mathrm{FV}=0$, but, if we wish to include a balloon payment with the last payment of the annuity, FV will be nonzero and automatically included in the relationship. When computing the future value of an annuity, we can either set $\mathrm{PV}=0$ or assume an initial payment was made and set PV equal to this amount. Many such instances arise in which this extension to six variables is useful in problem solving.

With this discussion in mind, we have the following six situations. For illustrative purposes we interpret each situation in terms of amortization (paying off a present debt) and as a sinking fund (accumulating a future sum). Any particular interpretation depends on the sign of PV, PMT, and FV and on what side of the transaction you are. For example, if you have an $\$ 80,000$ mortgage with $\$ 875$ payments and a balloon payment of $\$ 10,000$, then from your point of view the transaction looks like this:

| PV (INFLOW) ...........: | $\$ 80,000$ |
| :--- | ---: |
| PMT (OUTFLOW) $\ldots \ldots$ : | $-\$ 875$ |
| FV (OUTFLOW) . . . . . : | $-\$ 10,000$ |

but from the lender's side it looks like this:

| PV (OUTFLOW) $\ldots \ldots \ldots$. | $-\$ 80,000$ |
| :--- | ---: |
| PMT (INFLOW) $\ldots \ldots$ : | $\$ 875$ |
| FV (INFLOW) $\ldots \ldots \ldots$. . . . | $\$ 10,000$. |

The absolute value (the value without regard to algebraic sign) of any computation made with these two sets of inputs will be the same. The signs, however, will be reversed for PV, PMT, and FV.

## 1. FREQUENCY OF COMPOUNDING

The equation:

$$
\mathrm{F}=\mathrm{f}(\mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{PMT}, \mathrm{FV})
$$

ANSWERS: What is the number of payments per year given a total of N payments of amount PMT that pays off a debt PV, assuming a balloon payment FV and a discount rate I?
ANSWERS: What is the number of payments per year given a
total of N payments of amount PMT that will accumulate a sum FV, assuming an initial deposit PV and an interest rate I?

## 2. TIME TO MATURITY

The equation:

$$
\mathrm{N}=\mathrm{f}(\mathrm{~F}, \mathrm{I}, \mathrm{PV}, \mathrm{PMT}, \mathrm{FV})
$$

ANSWERS: What is the number of payments required to pay off a present debt PV with F payments a year of amount PMT at an annual interest rate I, assuming a balloon payment FV added to the final payment?
ANSWERS: What is the number of payments required to accumulate a sum FV with F payments a year in the amount PMT at an annual interest rate I , assuming an initial deposit PV?

## 3. ANNUAL INTEREST OR GROWTH RATE

The equation:

$$
\mathrm{I}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{PV}, \mathrm{PMT}, \mathrm{FV})
$$

ANSWERS: What interest rate will pay off a present debt PV in N periods with F payments per year in the amount PMT, assuming a balloon payment FV?
ANSWERS: What interest rate will accumulate a sum FV in N periods with F payments per year in the amount PMT, assuming an initial deposit PV ?

## 4. DISCOUNTING A FUTURE VALUE TO A PRESENT VALUE The equation:

$$
\mathrm{PV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PMT}, \mathrm{FV})
$$

ANSWERS: What is the present lump sum equivalent of N payments in the amount PMT with F payments made per year discounted at an annual rate $I$, assuming a balloon payment FV?
ANSWERS: What initial deposit must be made to accumulate a sum FV with N payments in the amount PMT with F payments made per year at an annual rate I?

## 5. PERIODIC PAYMENT OF AN ANNUITY

The equation:

$$
\mathrm{PMT}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{FV})
$$

ANSWERS: What payment is required to pay off a present debt PV with N payments making F payments per year and a balloon payment FV at an annual interest rate I?
ANSWERS: What payment is required to accumulate a sum FV with N payments, making F payments per year with an initial deposit PV at an annual rate I?

## 6. COMPOUNDING A PRESENT VALUE TO A FUTURE VALUE

The equation:

$$
\mathrm{FV}=\mathrm{f}(\mathrm{~F}, \mathrm{~N}, \mathrm{I}, \mathrm{PV}, \mathrm{PMT})
$$

ANSWERS: What balloon payment is required to pay off a present debt PV with N payments, making F payments per year in the amount PMT at an annual rate I?
ANSWERS: What future sum will be accumulated with N payments, making F payments per year in the amount PMT with an initial deposit PV at an annual rate I?

### 4.2. ORDANN: AN ORDINARY ANNUITY CALCULATOR

This calculator solves all the various permutations of the ordinary annuity problem which occur in many common business and finance situations. A typical screen is displayed in Table 4.1.

The input column for this calculator is the column of six cells, with numerical entries, at the top and to the right of the middle of the screen. The particular problem solved on this screen is the following: Compute the monthly payment of a $\$ 100,000,30$-year mortgage with an annual interest rate of $18 \%$. The answer is $-\$ 1,507.09$. The value is negative because it is a cash outflow.

This is an extremely powerful and flexible calculator that deserves careful study, both in its use and its construction. The examples given
next will illustrate applications and test your understanding of ordinary annuities.

### 4.3. HOW TO USE ORDANN: AN ORDINARY ANNUITY CALCULATOR

The basic concept in using all of the compound growth and annuity calculators is very simple. First, the given data is entered in the input column, then the single variable that is to be calculated is entered in the remaining input cell. The manual recalculation command is invoked, and the results are computed and posted to the specified table. Follow the step-by-step procedure in Fig. 4-1. The corresponding screen is displayed in Table 4-1.


## Table 4-1

## Helpful Hints on Using ORDANN

A. On each of the compound growth and annuity calculators, a pointer < is used to flag the calculated value in the input column. If an error message \#VALUE appears in some of the output tables, you may

## ORDANN: AN ORDINARY ANNUITY CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) ORDANN loaded and ready to use
(2) Manual recalculation set
(3) Cursor at POST TO TABLE cell, R2C10

PROBLEM DEFINITION: Compute the payment amount and post results to Table 2 with the following input.

$$
\begin{aligned}
\mathrm{F} & =12 \\
\mathrm{~N} & =360 \\
\mathrm{I} & =18 \% \\
\mathrm{PV} & =100000 \\
\mathrm{FV} & =0
\end{aligned}
$$



Fig. 4-1

## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN



Fig. 4-1 - cont.
have attempted to compute more than one value in the input column and there will be more than one pointer displayed there. For accurate results, you must enter only one variable in the input column and there will be only one pointer in the input column to reflect this condition.
B. Standard accounting practices regarding the sign of cash flows are observed on all of the compound growth and annuity calculators. If you obtain unreasonable answers, you may have entered a present value, a payment, or a future value with a sign inconsistent with your intentions. Assume the position of either the creditor or debtor, and then consistently give the correct sign to each cash value. Outflows are negative and inflows are positive.
C. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.

### 4.4. EXAMPLES USING ORDANN

NOTE: Work the first few problems of these examples, then refer to the six situations for ordinary annuities discussed in the introduction to this chapter, and make up your own examples for each case. Then, continue with the following examples. On completion, you will have a thorough knowledge of the concepts involved and should be ready to tackle an analysis of some of the new financing options that are becoming increasingly common.

1. Compute the payment on the example just illustrated, but assume the life of the mortgage is 25 years instead of 30 years, with a balloon payment of $\$ 10,000$. Post your results to TABLE 3 for use in the next problem.
2. Compute the total interest paid on the two loans discussed in example 1. The formula is ACCUMULATED INTEREST $=\mathrm{CV}-$ PV. In the first instance the cash value will be $\mathrm{N}^{*} \mathrm{PMT}$, and, in the second, it will be $\mathrm{N}^{*} \mathrm{PMT}+\mathrm{FV}$ because there is a balloon amount. Don't use pencil and paper, one of the cells in the input column can be used as a calculator. Enter the numerical values and arithmetic operations such as $12 * 25$ into the cell and press [! The result, " 300 ," will appear in the same cell.
3. You are paying $\$ 620$ a month on a home mortgage at $12.5 \%$ and have 65 payments left. What is the balance due on your loan? (Hint: The balance due on any cash flow is equal to the present value of the remaining payments, discounted at the appropriate rate.)
4. You owe a debt of $\$ 1,000$ to be fully paid in one year with monthly payments at $16 \%$. What are your payments?
5. You are paying off $\$ 1,000$ at $14.5 \%$ with $\$ 100$ monthly payments. How long must you pay? What is your last payment? (Hint: Be sure to enter values with an appropriate sign. Compute N. It will not be a whole number. Since it is impossible to make a fractional payment,
round N down to a whole number and enter that value as N . Then move the cursor to the FV cell and compute the balloon payment. Add this to your last payment of $\$ 100$ and you will have the complete solution.)
6. You have a 30 -year, $\$ 80,000$ mortgage at $15.25 \%$. If you make double payments for the first 5 years, how long will the mortgage run? Make an intuitive guess before you try to solve it. (Hint: There are several ways to do this. One approach is: Compute the monthly payment, then calculate how much of the $\$ 80,000$ five years of this payment will pay; i.e., compute the present value of 5 years at regular payments, then subtract this from $\$ 80,000$. This will be your new present value; now, simply compute N , the number of payments needed to pay off this new balance. This will be your answer. You're going to have to think about this one for a while, but the lesson is well worth the effort.)
7. You will borrow $\$ 20,000$ at $18 \%$ with quarterly payments. Your options are a loan for 5,7 , or 10 years. What will be your payments and the total interest paid in each case?

### 4.5. ANNDUE: AN ANNUITY DUE CALCULATOR

The screen design and structure of ANNDUE is exactly the same as ORDANN. For this reason, the most important aspect of this section is the examples which should be studied carefully. Although the structure is the same, the applications of annuities due are usually quite different than for ordinary annuities. A typical screen is displayed in Table 4-1.

### 4.6. HOW TO USE ANNDUE: AN ANNUITY DUE CALCULATOR

The basic concept in using all of the compound growth and annuity calculators is very simple. First, the given data is entered in the input column, then the single variable that is to be calculated is entered in the remaining input cell. The manual recalculation command is invoked, and the results are computed and posted to the specified table.

Follow the step-by-step procedure in Fig. 4-2. The corresponding screen is displayed in Table 4-2.

## Helpful Hints on Using ANNDUE

A. On each of the compound growth and annuity calculators, a pointer < is used to flag the calculated value in the input column. If an error message \#VALUE appears in some of the output tables, you may have attempted to compute more than one value in the input column and there will be more than one pointer displayed there. For accurate


Table 4-2
results, you must enter only one variable in the input column and there will be only one pointer in the input column to reflect this condition.
B. Standard accounting practices regarding the sign of cash flows are observed on all of the compound growth and annuity calculators. If you obtain unreasonable answers, you may have entered a present

## ANNDUE: AN ANNUITY DUE CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

## INITIAL CONDITIONS:

(1) ANNDUE Loaded and ready to use
(2) Manual recalculation set
(3) Cursor at POST TO TABLE cell, R2C10

PROBLEM DEFINITION: Compute the future value and post results to Table 1 with the following input.

$$
\begin{aligned}
\mathrm{F} & =12 \\
\mathrm{~N} & =60 \\
\mathrm{I} & =12 \% \\
\mathrm{PV} & =0 \\
\text { PMT } & =200
\end{aligned}
$$

## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN



Fig. 4-2


Fig. 4-2 - cont.
value, a payment, or a future value with a sign inconsistent with your intentions. Assume the position of either the creditor or debtor, and then consistently give the correct sign to each cash value. Outflows are negative and inflows are positive.
C. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.

### 4.7. EXAMPLES USING ANNDUE

1. In the example previously given, suppose you made an initial deposit of $\$ 1,000$ in addition to the $\$ 200$ payments. What would your balance be in this case? Post your results to TABLE 2.
2. You anticipate that in 5 years you will need $\$ 50,000$ to replace a high-speed printer in your print shop. If you make quarterly payments
into a sinking fund which yields $17.25 \%$, what will these payments be?
3. How long will it take $\$ 100$ monthly payments at $12 \%$ to accumulate $\$ 5,000$ ?
4. Your oldest child will start college in 8 years. At that time, you wish to provide her with a stipend of $\$ 600$ a month for 4 years to be paid at the beginning of each month. If you begin saving immediately with an existing account that earns $16.5 \%$ compounded monthly, what payments must you make? (Hint: Work backwards in two stages. First, compute the present value of the stipend, then let that value be the future value for your savings to compute the payments for the 8 -year period.)
5. You are 37 years old and expect to retire at age 55, manage your investments, and enjoy life. At that time you would like to have a special "buffer" account which would provide you with a minimum of $\$ 1,000$ a month for 10 years. If you can earn an average of $12 \%$ on your savings, what monthly payment must you make into your account until age 55 to achieve your goal?

### 4.8. CANN: A CONTINUOUS ANNUITY CALCULATOR

The screen design and structure of CANN is exactly the same as the other two annuity calculators in this series, ORDANN and ANNDUE. The formulas, however, are based on the continuous growth equation discussed in the section on CCOMP:

$$
\mathrm{FV}=\mathrm{PV} \exp (\mathrm{IT}) ;
$$

whereas, the other two annuity calculators are based on the compound growth equation discussed in the section on COMP:

$$
\mathrm{FV}=\mathrm{PV}(1 / \mathrm{P})^{\mathrm{N}}
$$

A typical screen is displayed in Table 4-3.

### 4.9. HOW TO USE CANN: A CONTINUOUS ANNUITY CALCULATOR

The basic concept in using all of the compound growth and annuity calculators is very simple. First, the given data is entered in the input column, then the single variable that is to be calculated is entered in the remaining input cell. The manual recalculation command is invoked and the results are computed and posted to the specified table. Follow the step-by-step procedure in Fig. 4-3. The corresponding screen is displayed in Table 4-3.

## Helpful Hints on Using CANN

A. On each of the compound growth and annuity calculators, a pointer < is used to flag the calculated value in the input column. If an error message \#VALUE appears in some of the output tables, you may


Table 4-3

CANN: A CONTINUOUS ANNUITY CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

## INITIAL CONDITIONS:

(1) CANN loaded and ready to use
(2) Manual recalculation set
(3) Cursor at POST TO TABLE cell, R2C10

PROBLEM DEFINITION: Compute the future value and post results to Table 1 with the following input.

$$
\begin{aligned}
\mathrm{F} & =12 \\
\mathrm{~N} & =12 \\
\mathrm{I} & =16 \% \\
\mathrm{PV} & =0 \\
\text { PMT } & =5000
\end{aligned}
$$

## RESULT OR RESPONSE <br> ACTION ON KEYBOARD ON SCREEN



Fig. 4-3


Fig. 4-3 - cont.
have attempted to compute more than one value in the input column and there will be more than one pointer displayed there. For accurate results, you must enter only one variable in the input column and there will be only one pointer in the input column to reflect this condition.
B. Standard accounting practices regarding the sign of cash flows are observed on all of the compound growth and annuity calculators. If you obtain unreasonable answers, you may have entered a present value, a payment, or a future value with a sign inconsistent with your intentions. Assume the position of either the creditor or debtor, and
then consistently give the correct sign to each cash value. Outflows are negative and inflows are positive.
C. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.

### 4.10. EXAMPLES USING CANN

1. A vending machine is predicted to generate a continuous income of $\$ 2,000$ per month for a period of 2 years before it has to be replaced. Discounting at the required rate of return of $24 \%$, what should you be willing to pay for such a machine?
2. Suppose a retail store nets an average of $\$ 400$ a day, seven days a week, compute the present value of this profit stream discounted at $16 \%$ for one year, assuming 360 days of operations.
3. Solve problem no. 2 using the ANNDUE calculator by inputting $\$ 400$ for the payment, 360 for $\mathrm{F}, 360$ for N , and $16 \%$ for the annual discount rate. This should give you a better feel for the relationship between these two calculators.

### 4.11. AMORT: AN AMORTIZATION SCHEDULE CALCULATOR

The purpose of this worksheet is to print out an annual monthly amortization schedule given the following information:

```
PROJECT NAME
LOAN AMOUNT
BALLOON PAYMENT
ANNUAL INTEREST RATE
TERM OF LOAN
BEGINNING DATE OF LOAN
BEGINNING DATE OF SCHEDULE TO BE COMPUTED
```

A typical input screen is displayed in Table 4-4.

### 4.12. HOW TO USE AMORT

A sample problem in using AMORT is illustrated in the walkthrough in Fig. 4-4. When you have completed the last step of this walkthrough, it will take a short time for the computation to be completed. The status of the calculation is displayed in the message line where it says "Cells to recalculate:"; when this cell reads zero, the computation

| AMORTIZATION SCHEDULE ANNUAL SUMMARY |  |  | BOAT LOAN |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 6000.00 |  |  |  |
| BALLOON PAYMENT |  |  | 0.00 |  |  |  |
| ANNUAL INTEREST RATE |  |  | 16.00\% |  | 1.33\% | \% PER MONTH |
| TERM OF LOAN |  |  | 5 YRS |  | 0 | MONTHS |
| BEGINNING BALANCE. . . . |  |  | 4585.54 |  | 19 | PMTS MADE |
| YEAR | MONTH | PAYMENT | PRINCIPAL | INTEREST |  | END BALANCE |
| 1983 | 1 | 145.91 | 84.77 | 61.14 |  | 4500.77 |
| 83 | 2 | 145.91 | 85.90 | 60.01 |  | 4414.87 |
| 83 | 3 | 145.91 | 87.05 | 58.86 |  | 4327.82 |
| 83 | 4 | 145.91 | 88.21 | 57.70 |  | 4239.62 |
| 83 | 5 | 145.91 | 89.38 | 56.53 |  | 4150.24 |
| 83 | 6 | 145.91 | 90.57 | 55.34 |  | 4059.66 |
| 83 | 7 | 145.91 | 91.78 | 54.13 |  | 3967.88 |
| 83 | 8 | 145.91 | 93.00 | 52.91 |  | 3874.88 |
| 83 | 9 | 145.91 | 94.24 | 51.67 |  | 3780.63 |
| 83 | 10 | 145.91 | 95.50 | 50.41 |  | 3685.13 |
| 83 | 11 | 145.91 | 96.77 | 49.14 |  | 3588.36 |
| 83 | 12 | 145.91 | 98.07 | 47.84 |  | 3490.29 |
| ANNUAL | L TOTAL | . . . 1750.92 | 1095.25 | 655.67 |  |  |
| TOTAL | TO DATE | .... 4523.21 | 2509.71 | 2013.50 |  |  |

Table 4-4
is finished. The completed schedule is displayed in Table 4-5. The values displayed may vary a few cents from published amortization schedules because of differences which accumulate in rounding errors, due to the use of different rounding techniques.

## Printing Out Your Amortization Schedule

1. As soon as the worksheet has been calculated, print the results from rows 24 through 51 between columns 1 and 5. Proceed as follows.
Press: $\mathbf{P}$ and $\mathbf{O}$
for the Print Options Command. The command line will read:
 with the cursor in the area field.
2. We now wish to specify the area of the sheet to be printed. This is R24:51C1:5 (rows 24 through 51 and columns 1 through 5).

## Press: R, 2, 4, ㄹ, 5, 1, C, 1, B, 5, and RETURN

3. We will now set the margins for your printout.

## Press: M

The command line will read:


This worksheet is set for an 80 -column printout, so we will set the left margin at 1 and arbitrarily set the top margin at 4 . The print width must be least 80 . The area to be printed is 28 lines long so, for good measure, let's make this parameter 30 . That will make the page length 34 , since the top margin is 4 . We are now ready to enter these values in the appropriate fields.
4. The cursor is in the left margin field.

Press: 1 and the retype key (left arrow key).
5. Now the cursor is in the top margin field.

Press: 4 and the retype key.
6. You should now be in the print width field. If the setting is already at 80 , hit the retype key to go to the next field; otherwise,
Press: 8, 0, and the retype key.
7. Now enter 30 in the print length field.

Press: 3, 0, and the retype key.

## AMORT: AMORTIZATION CALCULATOR EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) AMORT loaded and ready to use
(2) Manual recalculation set
(3) Cursor at PROJECT NAME cell, R7C3

PROBLEM DEFINITION: Generate an amortization schedule with the following input:

| Project name | BOAT LOAN |
| :---: | :---: |
| Loan amount | \$6,000 |
| Balloon payment | 0 |
| Annual interest rate | 16\% |
| Term of loan | 60 months |
| Beginning date of loan | June 1981 |
| Beginning date of schedul | January 1983 |

## RESULT OR RESPONSE ON SCREEN



Fig. 4-4

## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN



Fig. 4-4 - cont.
8. You are now in the page length field, the last field entry for this command.

## Press: 3, 4, and RETURN.

9. Upon hitting RETURN in the last step, the command line changed to:

PRINT: Printer File Margins Options


Table 4-5

| AMORTIZATION SCHEDULE ANNUAL SUMMARY |  |  | BOAT LOAN |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LOAN AMOUNT |  |  | 6000.00 |  |  |  |
| BALLOON PAYMENT |  |  | 0.00 |  |  |  |
| ANNUAL INTEREST RATE |  |  | 16.00\% |  | 1.33\% PER MONTH |  |
| TERM OF LOAN |  |  | 5 YRS |  | 0 | MONTHS |
| BEGINNING BALANCE. |  |  | 4585.54 |  | 19 | PMTS MADE |
| YEAR | MONTH | PAYMENT | PRINCIPAL | INTEREST |  | END BALANCE |
| 1983 | 1 | 145.91 | 84.77 | 61.14 |  | 4500.77 |
| 83 | 2 | 145.91 | 85.90 | 60.01 |  | 4414.87 |
| 83 | 3 | 145.91 | 87.05 | 58.86 |  | 4327.82 |
| 83 | 4 | 145.91 | 88.21 | 57.70 |  | 4239.62 |
| 83 | 5 | 145.91 | 89.38 | 56.53 |  | 4150.24 |
| 83 | 6 | 145.91 | 90.57 | 55.34 |  | 4059.66 |
| 83 | 7 | 145.91 | 91.78 | 54.13 |  | 3967.88 |
| 83 | 8 | 145.91 | 93.00 | 52.91 |  | 3874.88 |
| 83 | 9 | 145.91 | 94.24 | 51.67 |  | 3780.63 |
| 83 | 10 | 145.91 | 95.50 | 50.41 |  | 3685.13 |
| 83 | 11 | 145.91 | 96.77 | 49.14 |  | 3588.36 |
| 83 | 12 | 145.91 | 98.07 | 47.84 |  | 3490.29 |
| ANNUAL | L TOTAL | . . 1750.92 | 1095.25 | 655.67 |  |  |
| TOTAL | TO DATE | . . 4523.21 | 2509.71 | 2013.50 |  |  |

Table 4-6
with the cursor highlighting "Printer". The last step is to transfer the screen output to the printer; to do this:

Press: RETURN.
Compare your printout to the one displayed in Table 4-6.

### 4.13. EXAMPLES USING AMORT

The best way to familiarize yourself with this calculator is to make up your own examples and experiment with setting the printing options to get a desirable printout. You might try printing out several successive years of the example just discussed.

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## Chapter 5

## Discounted Cash Flow Analysis: Uneven Cash Flows

### 5.1. NPV: A NET PRESENT VALUE CALCULATOR



Discounted cash flow analysis and the concept of net present value were developed to provide an accurate measure of cash flows over time. When, say, your great grandfather made an investment, it was reasonable for him to forecast a profit stream and compute how long it would take to recover his money. This can still be done today, but, if the time period is very long at all, the high interest rates and inflation we are now living with will seriously distort such a straightforward approach. The following example will illustrate the concept of present value which overcomes the problem of equating a lump sum value to a stream of receipts.

Suppose you were offered your choice of the following income streams, which would you choose?

|  | A | B |
| :--- | :---: | :---: |
| 1st year | $\$ 1,000$ | $\$ 3,000$ |
| 2nd year | $\$ 2,000$ | $\$ 2,000$ |
| 3rd year | $\$ 3,000$ | $\$ 1,000$ |

In each case, you will get the same amount of cash, $\$ 6,000$. But you should agree that $B$ is more valuable. The real question is how much more valuable is $B$ than $A$ ? Or, put another way, how much would you be willing to pay right now for each of these cash flows?

One way to determine this is to compute the present value of each of the cash flows at an agreed on discount rate, and assign that as the current value of the cash flow. At $18 \%$ we have:

$$
\text { A: } \$ 4,109.72
$$

B: $\$ 4,587.37$.

Another way to interpret this is to suppose you deposit $\$ 4,109.72$ in an account that earns $18 \%$, compounded annually. At the end of the first year you could withdraw $\$ 1,000$, at the end of the second year you could withdraw $\$ 2,000$, and at the end of the third year you could withdraw $\$ 3,000$, and your account balance would be zero. The same reasoning applies to $B$.

The net present value of a cash flow is defined to be the present value of the cash flow less the initial cost of the project which generated the profit stream. This worksheet computes the net present value of a cash flow, given the cash values and the discount rate.

### 5.2. HOW TO USE NPV

A column of cash flow values and a discount rate are the input for this worksheet. A single value is the output: the net present value of the cash flow entered, discounted at the given rate. If the cash flow values are entered as yearly amounts, then the discount rate is an annual rate. If they are monthly amounts, the discount rate is a monthly rate; i.e., the annual rate divided by 12 . A similar convention is used for quarter or semi-annual periods.

Standard accounting conventions with regard to the sign of cash values are observed in entering the cash flow values. (Cash outflows are negative, cash inflows are positive.) The flow diagram in Fig. 5-1 walks you through a typical problem using NPV. Table 5-1 displays what your screen should look like after you have completed this problem. Note that the windows have been removed in this screen to show you the actual layout of the calculator on the worksheet.

## Helpful Hints on Using NPV

A. If a series of cash flows are entered, and during some periods the cash flow is zero, then a zero value must be entered for these periods. If you simply leave the space blank, your answer will not be correct. You may leave all of the spaces blank after the last cash value entered; although your answer will still be correct if zero values are in these cells.

## NPV: A NET PRESENT VALUE CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) NPV loaded and ready to use
(1) Manual recalculation set
(3) Cursor at CASH FLOW 0 cell, R2C2

PROBLEM DEFINITION: Compute the net present value of the following cash flow discounting at $18 \%$ :

Investment . . . . . . . . . . . . . . . . . . . $\$ 4,000$
Period 1 . . . . . . . . . . . . . . . . . . . . . . $\$ 1,000$
Period 2 . . . . . . . . . . . . . . . . . . . . . . . $\$ 2,000$
Period 3 . . . . . . . . . . . . . . . . . . . . . . \$3,000

## RESULT OR RESPONSE ON SCREEN



## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN

|  | Enters '18\%' in the DISCOUNT RATE cell and recalculates the worksheet. Displays ' $\$ 109.72$ ' in the NPV cell. |
| :---: | :---: |

Fig. 5-1 - cont.


Table 5-1
B. Sometimes it is necessary to clear the input column when, for example, fewer values are to be entered than already are in the column. Fig. 5-2 illustrates how this is done.
C. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
D. You may wish to print out this particular worksheet so you have a
hardcopy for your own records. Detailed instructions on printing procedures are given in Section 4.12 which discusses how to print amortization schedules. The only difference will be the begin-print and end-print coordinates (cells) entered.

## CLEARING AN ENTRY COLUMN

## INITIAL CONDITIONS:

(1) One of the calculators loaded and ready to use
(2) Cursor located at the top of column to be cleared

PROBLEM DEFINITION: Clear the top six cells of an input column of existing entries with the COPY DOWN command.

## RESULT OR RESPONSE ON SCREEN



Fig. 5-2

### 5.3. EXAMPLES USING NPV

1. In the preceding example we discounted at $18 \%$. Work the same problem discounting at $16 \%$ and $20 \%$, and compare your results.
2. Suppose you have a project with an initial investment of $\$ 10,000$, another cash outlay of $\$ 15,000$ at the end of the first year, then an expected net profit of $\$ 8,000$ per year for the second through the seventh years. If you require a $20 \%$ yield, is this an acceptable proj-
ect? Note here that the first two cash flows are negative and the remaining six are positive.
3. You may invest in a project that will return $\$ 1,000$ a month for the first year and $\$ 2,000$ a month for the second year. The total cash value received will be $\$ 36,000$. Your investment would be $\$ 30,000$. If you require an $18 \%$ return, would you consider this project? Note that your discount rate here will be a monthly, not an annual, rate. It is also convenient to use the "Copy Down" command to enter values that are identical in the input column.

### 5.4. IRR: INTERNAL RATE OF RETURN

The internal rate of return of a cash flow is defined to be the discount rate that makes the present value of the cash flow equal in value to the amount of the initial investment. Conceptually, it is a measure of the yield on an investment. The input required for IRR is a cash flow. All outflows are entered as negative values, and inflows, positive. The output is the internal rate of return as a percent.

### 5.5. HOW TO USE IRR: INTERNAL RATE OF RETURN

For this calculator, a column of cash flow values is the input. A single value is the output: the internal rate of return. If the cash flow values are entered as yearly amounts, the IRR is an annual rate. If they are monthly amounts, the IRR. is a monthly rate; i.e., the annual rate divided by 12. A similar convention is used for quarter or semi-annual periods.

Standard accounting conventions with regard to the sign of cash values are observed in entering the cash flow values. (Cash outflows are negative and cash inflows are positive.) The flow diagram in Fig. 5-3 walks you through a typical problem using IRR. Table 5-2 displays what your screen should look like after you have completed this problem. Note that the windows have been removed in this screen to show you the actual layout of the calculator on the worksheet.

## IRR: AN INTERNAL RATE OF RETURN CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) IRR loaded and ready to use
(2) Manual recalculation set
(3) PAYMENT column cleared for input
(4) Cursor at CASH FLOW 0 cell, R2C2

PROBLEM DEFINITION: Compute the internal rate of return of the following cash flow:

| Investment | . \$4,000 |
| :---: | :---: |
| Period 1 | . \$1,000 |
| Period 2 | . \$2,000 |
| Period 3 | . \$3,000 |

## RESULT OR RESPONSE ON SCREEN



Fig. 5-3


## Table 5-2

## Helpful Hints on Using IRR

A. If a series of cash flows is entered, and during some periods the cash flow is zero, then a zero value must be entered for these periods. If you simply leave the space blank, your answer will not be correct. You may leave all of the spaces blank after the last cash value entered; although your answer will still be correct if you have zero values in these cells.
B. It is sometimes necessary to clear the input column when, for example, fewer values are to be entered than already are in the column Fig. 5-2 illustrates how this is done.
C. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
D. You may wish to print out this particular worksheet so you have a hardcopy for your own records. Detailed instructions on printing
procedures are given in Section 4.12, which discusses how to print amortization schedules.

### 5.6. EXAMPLES USING IRR

1. An important value of an IRR calculator such as this one is that values can be changed and recalculated very easily. This gives you an opportunity to experiment with numerical values, and get a feel for the impact of a change in income or outlay on the yield of an investment. Sometimes a small change in the cash flow can make a surprisingly large change in the IRR which would indicate, in such a case, that IRR is not as sound a measure of yield as might be thought. Experiment with the values in the previous example to see what they do to the IRR.
2. Compute the IRR of the cash flow described in problem 2 in the examples given in Section 5.3.
3. Compute the IRR of the cash flow described in problem 3 in the examples given in the last section (Section 5.3). When the IRR appears, multiply it by 12 in a blank cell in the input column. For example, to multiply 2 times 12 , write the expression $2^{*} 12$ and press [!. You can calculate the value of any arithmetic expression in a blank cell in this way.

### 5.7. FMRR: A FINANCIAL MANAGEMENT RATE OF RETURN CALCULATOR

The financial management rate of return is a measure of the yield on an investment that is more sensitive to the actual conditions under which a project is undertaken than is the internal rate of return method. An example will illustrate this. Suppose you invest $\$ 1,000$ and 4 years later receive $\$ 2,000$. Your internal rate of return is $18.92 \%$, and, in this case, your financial management rate of return is also $18.92 \%$.

Now let's change the conditions. Suppose you invest $\$ 1,000$, receive $\$ 1,000$ at the end of the second year and $\$ 1,000$ at the end of the fourth year.

| 0 | $-\$ 1,000$ |
| ---: | ---: |
| 1 | 0 |
| 2 | $\$ 1,000$ |
| 3 | 0 |
| 4 | $\$ 1,000$ |

The internal rate of return here is $27.20 \%$. What we are trying to measure is your yield during the 4 -year period. In this particular example, you got $\$ 1,000$ after 2 years. What did you do with this money? The IRR method assumes you reinvested it at $27.20 \%$ until the end of the 4-year period (compound it to the end of the period, add the other $\$ 1,000$ and then compute the PV to check this). This is not necessarily a reasonable assumption. Suppose your reinvestment rate was only $18 \%$, then at the end of the period this amount, $\$ 1,000$, would have grown to $\$ 1,392.40$, and your total return in dollars would be $\$ 2,392.40$. With $\$ 1,000$ at PV and $\$ 2,392.40$ as FV, this gives you a yield of $24.37 \%$, about $3 \%$ lower than the IRR.

If longer periods are involved, this discrepancy is magnified considerably. In addition to the reinvestment rate, we also consider our cost of capital in computing the financial management rate of return. If we have negative cash flows during the course of a project, we assume that a cost is associated with these outflows which is measured by a discount rate, our cost of borrowing money for instance. Our procedure, then, is to separate our cash flow into two cash flows, one consisting of all outflows and the other, all inflows. We then compute the present value of the outflows to the beginning of the project and the future value of the inflows to the end of the project. The interest rate that equates these two values is the FMRR.

### 5.8. HOW TO USE FMRR: FINANCIAL MANAGEMENT RATE OF RETURN

For this calculator, a column of cash flow values, a reinvestment rate, a discount rate and the number of periods over which the investment is made are the inputs. A single value is the output: the financial management rate of return of the cash flow. If the cash flow values are entered as yearly amounts, then all rates are annual rates. If they are
monthly amounts, then the rates are monthly rates, i.e., the annual rate divided by 12 . A similar convention is used for quarter or semiannual periods.

Standard accounting conventions with regard to the sign of cash values are observed in entering the cash flow values. (Cash outflows are negative and cash inflows are positive.) The flow diagram shown in Fig. 5-5 walks you through a typical problem using FMRR. Table 5-3 displays what your screen should look like after you have completed this problem. Note that the windows have been removed in this screen to show you the actual layout of the calculator on the worksheet.


Table 5-3

## Helpful Hints on Using FMRR

A. If a series of cash flows is entered, and during some periods the cash flow is zero, then you must enter a zero value for these periods. If you simply leave the space blank your answer will not be correct. You may leave all of the spaces blank after the last cash value entered; al-

## FMRR: FINANCIAL MANAGEMENT RATE OF RETURN CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) NPV loaded and ready to use
(2) Manual recalculation set
(3) PAYMENT column cleared for input
(4) Cursor at CASH FLOW 0 cell, R6C2

PROBLEM DEFINITION: Compute the financial management rate of return of the following cash flow. Use a discount rate of $16 \%$, a reinvestment rate of $21 \%$, and a period of three years.

| Investment | 4,000 |
| :---: | :---: |
| Period 1 | \$1,000 |
| Period 2 | \$2,000 |
| Period 3 | \$3,000 |

## RESULT OR RESPONSE ON SCREEN

Enters '-4000' as cash flow 0 and moves the cursor to the next input cell.

Enters '1000' as cash flow 1 and moves the cursor to the next input cell.


Enters '2000' as cash flow 2 and moves the cursor to the next input cell.


Enters ' 3000 ' as cash flow 3 and moves the cursor to the REINVESTMENT RATE cell.


Enters '21\%' in the REINVESTMENT RATE cell and moves the cursor to the next input cell.

Fig. 5-4

## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN

| Press: 1, 6, \% and Ctrl-X $\text { Press: } \boldsymbol{\square}, \begin{aligned} & \text { or } \\ & \mathbf{1} \\ & \mathbf{6} \\ & \hline \end{aligned}$ | Enters ' $16 \%$ ' in the DISCOUNT RATE cell and moves the cursor to the next input cell. |
| :---: | :---: |
| $\downarrow$ | Enters ' 3 ' in the NUMBER OF PERIODS cell and recalculates the worksheet. Displays ' $19.84 \%$ ' in the FMRR cell. |
| Press: 3, RETURN and ! |  |

Fig. 5-4 - cont.
though your answer will still be correct if you have zero values in these cells.
B. It is sometimes necessary to clear the input column when, for example, fewer values are to be entered than already are in the column. Fig. 5-5 illustrates how this is done.
C. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
D. You may wish to print out this particular worksheet so that you have a hardcopy for your own records. Detailed instructions on printing procedures are given in Section 4.12 which discusses how to print amortization schedules.

### 5.9. EXAMPLES USING FMRR

1. In the preceding example, enter another value for the discount rate and notice what happens. As you will see it has no effect on the FMRR. The reason is that there is only one negative cash flow at the beginning of the project, and, no matter at what rate it is discounted, its present value will remain unchanged.
2. Work problem 2 from the examples in section 5.3 with a reinvest-

## CLEARING AN ENTRY COLUMN

## INITIAL CONDITIONS:

(1) One of the calculators loaded and ready to use
(2) Cursor located at the top of column to be cleared

PROBLEM DEFINITION: Clear the top six cells of an input column of existing entries with the COPY DOWN command.

## RESULT OR RESPONSE ON SCREEN

ACTION ON KEYBOARD


Fig. 5-5
ment rate of $20 \%$ and a discount rate of $16 \%$. Compare this value with the IRR for the same project.
3. Work problem No. 3 from the examples in Section 5.3 with a reinvesment rate of $18 \%$ and a discount rate of $12 \%$. Compare this value with the IRR.

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## Chapter 6

## Profit Planning Tools

### 6.1. BREAK: A BREAK-EVEN ANALYSIS CALCULATOR



Break-even analysis provides us with a model of the relationship between fixed costs, variable costs, and profits. Its simplicity, both conceptually and mathematically, is deceiving for it would seem that such a simple model could not explain very much. The truth is that its very simplicity allows us to take a global view of the operations of a firm, and see them as dynamically interacting systems. It provides us with a way of getting a handle on the uncertainties of planning, by exploring many alternatives for employing idle plant capacity, expanding production, changing pricing structures and policies, setting sales goals and advertising budgets, looking at alternative materials purchases, and even granting credit.

If you have ever done any break-even analysis, you know what this worksheet is all about; but the odds are it is not used as a working tool in many instances where it could be beneficial in the planning process. This worksheet is a classic example of the "what if" game for which electronic spreadsheets are so famous, and it is one of the easiest to use and to understand.

### 6.2. HOW TO USE BREAK

BREAK naturally divides itself into four sections, each of which will be discussed separately in the order they are presented on the worksheet. Table 6-1 shows an example of a typical screen.

## Selling Price, Unit Cost, and Margin

The first section is a calculator that relates three variables: the unit selling price (SEL), the unit cost (CST), and the contribution margin
(MAR). Given any two of these, the third can be computed using one of the following formulas:

$$
\begin{aligned}
& \text { MAR }=[\text { SEL }- \text { CST }] / \text { SEL } \\
& \text { CST }=[1-\mathrm{MAR}]^{*} \mathrm{SEL}, \\
& \text { SEL }=\mathrm{CST} /[1-\mathrm{MAR}] .
\end{aligned}
$$



## Table 6-1

## Fixed Costs

As soon as SEL, CST, and MAR are entered on the worksheet, the only other value necessary to compute the break-even point is fixed costs. This is entered in the first line of the second section of the worksheet. Three columns are provided to input alternative values. You might, for instance, look at the best and worst cases and at something in between.

## Desired Profit

To compute the sales required to support a given profit level, simply input your desired profit and recalculate the worksheet.

## Desired Profit Margin

Similarly, to compute the sales required to support a given profit margin, enter the margin and recalculate the worksheet.

The flow diagram shown in Fig. 6-1 walks you through a typical application of BREAK. The corresponding screen is displayed in Table 6-1.

## Helpful Hints on Using BREAK

A. Note that when you wish to enter a value such as $\$ 15,000.00$ in a cell with a dollar format, the dollar sign and the comma should not be entered. The period and the last two zeros may or may not be entered; however, the period is required if the last two zeros are entered. (In this case, you could enter " 15000 " or " 15000.00 ", and " $\$ 15000.00$ " would be displayed.)
B. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
C. You may wish to print out this particular worksheet so that you have a hardcopy for your own records. Detailed instructions on printing procedures are given in Section 4.12 which discusses how to print amortization schedules.

### 6.3. EXAMPLES USING BREAK

1. The Sandalwood Corporation manufactures a laser carved, wooden decoration for offices with a factory price of $\$ 8.00$ and a unit cost of $\$ 5.50$. They have a vacant plant with a potential of manufacturing 15,000 units per month, or 180,000 per year. Their annual fixed costs are estimated to be $\$ 250,000$. Based on this information, should Sandalwood pursue a detailed study of opening their unused plant for this project?
2. An alternative is to lease the facility for $\$ 120,000$ per year, an option that is currently viable. Enter this value into the desired profit row to see if the expansion plans could do better.

## BREAK: BREAK-EVEN ANALYSIS CALCULATOR

EXAMPLE OF INPUT AND OUTPUT PROCEDURES

## INITIAL CONDITIONS:

(1) BREAK loaded and ready to use
(2) Manual recalculation set
(3) Cursor at UNIT SELLING PRICE cell, R4C3

PROBLEM DEFINITION:
PART 1. Compute the UNIT COST given a UNIT SELLING PRICE of $\$ 40.00$ and a CONTRIBUTION MARGIN of $15 \%$.
PART 2. Compute the break-even point in dollars and units given fixed costs of $\$ 15,000, \$ 20,000$, and \$25,000.


Fig. 6-1

## RESULT OR RESPONSE ON SCREEN

PART 2


NOTE: The remaining two sections of this worksheet accept data and are recalculated in exactly the same way as demonstrated in the above example.

Fig. 6-1 - cont.
3. The Sandalwood marketing people feel a sales potential of $2,000,000$ units exists for their product next year, and a $5 \%$ market penetration is predicted with no increase in advertising. If management requires a minimum operating profit of $\$ 250,000$, is the project going to fly?
4. There is another kicker here, also. A \$2,000 monthly advertising
budget could increase the market penetration by another $2 \%$ to $3 \%$. Being conservative, you assume you can get the $2 \%$, giving you potential sales of $7 \%$ of $2,000,000$ units. Add the advertising budget to your fixed costs and see where this puts you.
5. Max in R\&D has just come up with a new laser device that he can build and install in time for start-up operations on this new project. It's going to cost $\$ 65,000$, and that is a hard figure. He also estimates it will take $\$ .50$ off the factory cost of each unit produced. The beauty of the new machine is it will increase production to 20,000 units per month. Now, rethink the last question in light of this new information.

As you may have concluded after working these examples, the variations on these themes are endless. Suppose you are looking at a new distributor and want to evaluate him as a credit risk, using quick and dirty break-even analysis. Plug in as fixed costs your potential losses (carrying costs for the account, plus administrative expenses to service the account, plus your actual losses based on your predictions of his average balance). This will give you a break-even figure. If his expected sales are well above this break-even point, you have a green light to spend some time building a worksheet which will give you a more detailed picture of a situation that looks promising.

Another possibility is that your unit costs may not be constant at all production levels. It may be more or less expensive to add a crew that can produce another 20,000 units per year. You then have to do your analysis in steps. If there are economies of scale, this is essential; and it is very easy to do with BREAK.

### 6.4. DEP: DEPRECIATION METHODS

The method used to compute depreciation has a direct impact on the tax environment of a firm over the life of an asset, since depreciation charges are deductible in computing federal income taxes. Accelerated methods of depreciation reduce the tax burden in the early years of the life of an asset.

This worksheet generates depreciation schedules using the three
principal methods of depreciation: straight-line, sum-of-the-yearsdigits, and double declining balance. Schedules for all three methods of depreciation are produced simultaneously with the input of the following data:

> Beginning date (MM/YYYY),
> Estimated useful life, Starting book value, and Salvage value.

The program automatically computes depreciation for a partial year if the beginning date is other than January. A typical printout is displayed in Table 6-2, with a beginning date of January, 1978. The same schedule is displayed in Table 6-3 with the beginning date of March, 1978 to illustrate this feature.

## Straight-Line Depreciation

The schedules displayed show that straight-line depreciation gives us a uniform annual depreciation charge of $\$ 3,000$. This is computed by dividing the difference between the cost of the item (the starting book value) and the salvage value by its estimated useful life.

## Sum-of-the-Years-Digits Depreciation

This method computes the annual depreciation allowance as a fraction of the depreciable cost of an asset (beginning book value less salvage value), which is $\$ 15,000$ in our example. Since the depreciable life is 5 years, the sum of the digits is:

$$
1+2+3+4+5=15
$$

Our computation is:

| 1st year | $(\$ 15,000)(5 / 15)=\$ 5,000$ |
| :--- | :--- |
| 2nd year | $(\$ 15,000)(4 / 15)=\$ 4,000$ |
| 3rd year | $(\$ 15,000)(3 / 15)=\$ 3,000$ |
| 4th year | $(\$ 15,000)(2 / 15)=\$ 2,000$ |
| 5th year | $(\$ 15,000)(1 / 15)=\$ 1,000$ |

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| DEP: DE | DEPRECIATION SCHEDULES |  |  |
| :---: | :---: | :---: | :---: |
| BEGINNING DATE |  |  |  |
| MONTH (MM). |  |  |  |
| YEAR (YYYY) |  |  |  |
| ESTIMATED USEFUL LIFE (YEARS) |  |  |  |
| STARTING BOOK VALUE (\$) |  |  |  |
| SALVAGE VALUE (\$) |  |  |  |
| STRAIGHT-LINE DEPRECIATION SCHEDULE |  |  |  |
| YEAR | ANNUAL DEP | ACCUMULATED DEP | BOOK VALUE |
| 1978 | \$3,000.00 | \$3,000.00 | \$12,300.00 |
| 1979 | \$3,000.00 | \$6,000.00 | \$9,300.00 |
| 1980 | \$3,000.00 | \$9,000.00 | \$6,300.00 |
| 1981 | \$3,000.00 | \$12,000.00 | \$3,300.00 |
| 1982 | \$3,000.00 | \$15,000.00 | \$300.00 |
| SUM-OF-THE-YEARS-DIGITS DEPRECIATION SCHEDULE |  |  |  |
| YEAR | ANNUAL DEP | ACCUMULATED DEP | BOOK VALUE |
| 1978 | \$5,000.00 | \$5,000.00 | \$10,300.00 |
| 1979 | \$4,000.00 | \$9,000.00 | \$6,300.00 |
| 1980 | \$3,000.00 | \$12,000.00 | \$3,300.00 |
| 1981 | \$2,000.00 | \$14,000.00 | \$1,300.00 |
| 1982 | \$1,000.00 | \$15,000.00 | \$300.00 |
| DECLINING BALANCE DEPRECIATION SCHEDULE |  |  |  |
| YEAR | ANNUAL DEP | ACCUMULATED DEP | BOOK VALUE |
| 1978 | \$6,120.00 | \$6,120.00 | \$9,180.00 |
| 1979 | \$3,672.00 | \$9,792.00 | \$5,508.00 |
| 1980 | \$2,203.20 | \$11,995.20 | \$3,304.80 |
| 1981 | \$1,321.92 | \$13,317.12 | \$1,982.88 |
| 1982 | \$793.15 | \$14,110.27 | \$1,189.73 |

## Table 6-2

## Double Declining Balance Depreciation

This is an accelerated method of depreciation which computes the annual depreciation allowance as a flat percentage of the book value of an asset. The rate of depreciation used is $2 / \mathrm{N}$, where N is the estimated useful life of the asset. In this case we have:

$$
2 / \mathrm{N}=2 / 5=0.40=40 \%
$$

The annual depreciation, thus, is computed as follows:

$$
\begin{array}{ll}
\text { 1st year } & (\$ 15,300)(.40)=\$ 6,120.00 \\
\text { 2nd year } & (\$ 9,180)(.40)=\$ 3,672.00 \\
\text { 3rd year } & (\$ 5,308)(.40)=\$ 2,203.20
\end{array}
$$

| DEP: DEPRECIATION SCHEDULES |  |  |  |
| :---: | :---: | :---: | :---: |
| BEGINNING DATE |  |  |  |
| MONTH (MM). |  |  | 3 |
| YEAR (YYYY) |  |  | 1978 |
| ESTIMATED USEFUL | LIFE (YEARS) |  | 5 |
| STARTING BOOK VAL | UE (\$) . . . . | .... | 15300 |
| SALVAGE VALUE (\$). |  |  | 300 |
| STRAIGHT-LINE DEPRECIATION SCHEDULE |  |  |  |
| YEAR | ANNUAL DEP | ACCUMULATED DEP | BOOK VALUE |
| 1978 | \$2,500.00 | \$2,500.00 | \$12,800.00 |
| 1979 | \$3,000.00 | \$5,500.00 | \$9,800.00 |
| 1980 | \$3,000.00 | \$8,500.00 | \$6,800.00 |
| 1981 | \$3,000.00 | \$11,500.00 | \$3,800.00 |
| 1982 | \$3,000.00 | \$14,500.00 | \$800.00 |
| 1983 | \$500.00 | \$15,000.00 | \$300.00 |
| SUM-OF-THE-YEARS-DIGITS DEPRECIATION SCHEDULE |  |  |  |
| YEAR | ANNUAL DEP | ACCUMULATED DEP | BOOK VALUE |
| 1978 | \$4,166.67 | \$4,166.67 | \$11,133.33 |
| 1979 | \$4,166.67 | \$8,333.33 | \$6,966.67 |
| 1980 | \$3,166.67 | \$11,500.00 | \$3,800.00 |
| 1981 | \$2,166.67 | \$13,666.67 | \$1,633.33 |
| 1982 | \$1,166.67 | \$14,833.33 | \$466.67 |
| 1983 | \$166.67 | \$15,000.00 | \$300.00 |
| DECLINING BALANCE DEPRECIATION SCHEDULE |  |  |  |
| YEAR | ANNUAL DEP | ACCUMULATED DEP | BOOK VALUE |
| 1978 | \$5,100.00 | \$5,100.00 | \$10,200.00 |
| 1979 | \$4,080.00 | \$9,180.00 | \$6,120.00 |
| 1980 | \$2,448.80 | \$11,628.)0 | \$3,672.00 |
| 1981 | \$1,468.80 | \$13,096.80 | \$2,203.20 |
| 1982 | \$881.28 | \$13,978.08 | \$1,321.92 |
| 1983 | \$528.77 | \$14,506.85 | \$793.15 |

Table 6-3

4th year
$(\$ 3,304)(.40)=\$ 1,321.92$
5th year
$(\$ 1,983)(.40)=\$ 793.15$.
Note that this rate is applied to the full book value; the salvage value is not deducted using this method.

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### 6.5. HOW TO USE DEP

The flow diagram in Fig. 6-2 walks you through a typical problem using DEP. Table 6-2 displays a printout of the results of this problem.

## Helpful Hints on Using DEP

A. The next step is to print out the schedules you have generated. The area to print for this particular schedule is $\mathrm{R} 1: 11 \mathrm{C} 1: 16$. The PRINT MARGINS are
left: 1
top: 4
print width: 80
print length: 12
page length: 15 .

These are accomplished with the Print command. Instructions for printing out a worksheet are detailed in Chapter 4, Section 4.12.

### 6.6. EXAMPLES USING DEP

1. You have just purchased a $\$ 60,000$ milling machine with a useful life of 10 years and a salvage value of $\$ 20,000$. Construct a set of depreciation schedules for this machine.
2. Print out the schedule you generated in problem no. 1, since it is a longer schedule you will have to adjust the print area accordingly.

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Weston, J. Fred and Eugene F. Brigham. Managerial Finance. Hinsdale, Illinois: The Dryden Press, 1978.

## DEP: DEPRECIATION SCHEDULE CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

## INITIAL CONDITIONS:

(1) DEP loaded and ready to use
(2) Manual recalculation set
(3) Cursor at BEGINNING MONTH cell, R5C3

PROBLEM DEFINITION: Generate depreciation schedules with the following input:

Beginning month

:1

Beginning year . . . . . . . . . . . . . . . : 1978
Estimated useful life . . . . . . . . . . : 5
Starting book value . . . . . . . . . . . $\$ 15,300$
Salvage value . . . . . . . . . . . . . . . :\$300

## RESULT OR RESPONSE <br> ACTION ON KEYBOARD ON SCREEN



## RESULT OR RESPONSE ON SCREEN

| Press: 3, 0, 0, RETURN <br> and $\boldsymbol{1}$ | Enters '300' in the <br> SALVAGE VALUE cell and <br> recalculates the worksheet. <br> See instructions in the text <br> for printing out the <br> schedules. |
| :--- | :--- |

Fig. 6-2 - cont.

## Chapter 7

## Basic Statistics and Forecasting

### 7.1. STAT: MEAN, STANDARD DEVIATION, AND VARIANCE


and

The basic purpose of statistics is to gather data and organize it into a form that gives a succinct overall picture of what that data represents. Suppose two sets of data points are given:

$$
\text { (A) } 0,1,2,3,4,5,6,7,8,9
$$

(B) $4,5,3,5,5,6,3,4,5,5$.

Both of these sets of data have a sum of 45 and, therefore, an arithmetic mean (average) of 4.5 . Most of us would agree that the number 4.5 is a fairly accurate representation of the set ( B ), since the numbers of this set cluster around this average, whereas 4.5 is not an accurate representation of the set $(\mathrm{A})$ simply because the numbers in this set are not clustered in the same way.

This example illustrates how a statistical measure, such as a mean, can be misleading, but, also, clearly demonstrates the need of some measure on how tightly clustered the data is around the mean in order to realistically interpret this particular statistic. One such measure is the range:

|  | MEAN | RANGE |
| :--- | :---: | :---: |
| (A) | 4.5 | 0 to 9 |
| (B) | 4.5 | 3 to 6. |

This is a good first approximation to a measure of how disperse our data set is, but a better one is often used. It is called the standard deviation:

MEAN
4.5
(A)
(B)
4.5

STANDARD DEVIATION
0.92
2.87

Roughly speaking, the standard deviation tells us the average distance a typical point in the data set is "off" from the mean. On the average, $75 \%$ of all the data points will lie within two standard deviations of the mean. If our data set is normally distributed in the classic bell curve, then, on the average, $68 \%$ of all data points will lie within one standard deviation of the mean.

Applying this first rule to the data set (B), $75 \%$ of the data points lie in the range:

$$
4.5-2(.92)=4.5-1.84=2.66
$$

to

$$
4.5+2(.92)=4.5+1.84=6.34
$$

In fact, $100 \%$ of the points happen to fall within this range. In general, you should be able to give 3 to 1 odds (a probability of $75 \%$ ) that any data point, randomly selected, will fall within two standard deviations of the mean. If the data set is normally distributed, you can give 2 to 1 odds (a probability of $68 \%$ ) that any randomly selected data point will fall within one standard deviation of the mean.

It sometimes is useful to express the standard deviation as a percentage of the mean. This is done by dividing the standard deviation by the mean and expressing the result as a percent. This number is called the coefficient of variation, and is a relative measure of dispersion; whereas, the standard deviation is an absolute measure of dispersion.

As an example, suppose, in six trials, a runner averages 4.90 minutes for a mile with a standard deviation of .12 minute.

Expressing this as a percent gives

$$
100(.12 / 4.90)=2.45 \%
$$

which is a relative measure of how consistent his performance is. Namely, $75 \%$ of the time he runs within $2.45 \%$ of his average training time.

The square of the standard deviation is called the variance. It, also, is computed on the STAT worksheet. All of these statistics are computed for data sets (A) and (B) on the two, typical worksheet screens shown in Tables 7-1 and 7-2.


Table 7-1


Table 7-2

Notice, on the screens just illustrated, two standard deviations are reported. One is called a sample standard deviation, the other, a population standard deviation. You may have noticed in both of the examples cited, the sample standard deviation is the larger of the two. The sample standard deviation is used to make predictions about a total population from a representative sampling. It always is larger because it must be a more conservative figure in order to ensure that it is a good estimator.

### 7.2. HOW TO USE STAT

The input for STAT is a set of data that is entered in a single column. A typical output is displayed in Table 7-1. Note that the windows have been removed in this screen to show you the actual layout of the calculator on the worksheet. The flow diagram in Fig. 7-1 walks you through a typical problem using STAT.

## Helpful Hints on Using STAT

A. It is sometimes necessary to clear the input column when, for instance, you wish to enter fewer values than already are entered in the column. Fig. 7-2 illustrates how this is done.
B. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
C. You may wish to print out this particular worksheet so that you have a hardcopy for your own records. Detailed instructions on printing procedures are given in Section 4.12 which discusses how to print amortization schedules.
D. This worksheet is very simple, yet powerful. It is set up to allow the entry of up to 200 data points in column 1. If you wish to extend this, you must move the output window to the right with the move command, move column 2 to where you want it, then move column 3 to the immediate right of it. Let's say you moved the output window to columns 4 and 5 so you could input data in columns 1, 2, and 3. You may enter up to 255 data points in each column; therefore, this setup would give you a maximum of 765 data points to play with. The

## STAT: STATISTICAL FUNCTION CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) STAT loaded and ready to use
(2) Manual recalculation set
(3) Entry column cleared for input
(4) Cursor at R1C1

PROBLEM DEFINITION: Compute the mean, standard deviation, coefficient of correlation, and standard deviation of the following set of data.

$$
12,15,13,15,11,17
$$

## RESULT OR RESPONSE ON SCREEN

## ACTION ON KEYBOARD



Fig. 7-1


Fig. 7-1 - cont.
200 data points were chosen as an uper limit simply because, in practice, it is a reasonable number to deal with, in many cases. The worksheet, also, is relatively fast computing with this number.

## CLEARING AN ENTRY COLUMN

INITIAL CONDITIONS:
(1) One of the calculators loaded and ready to use
(2) Cursor located at the top of column to be cleared

PROBLEM DEFINITION: Clear the top six cells of an input column of existing entries with the COPY DOWN command.

## ACTION ON KEYBOARD RESULT OR RESPONSE ON SCREEN



### 7.3. EXAMPLES USING STAT

1. Let's suppose you have taken a random sample of 100 guests at a hotel during a one-month period. You compute, with STAT, their average stay has been 2.4 days with a sample standard deviation of 1.5 days. We may assume this population is normally distributed, and, therefore, may conclude that $68 \%$ of the guests at this hotel stay between:

$$
2.4-1.5=0.90 \text { days }
$$

and

$$
2.4+3.5=3.9 \text { days }
$$

If the minimum stay at the hotel is one day, what percent of the guests stay four or more days?
2. The coefficient of variation is a measure of the consistency of the data in terms of how it clusters around the mean of the population. Suppose you are a credit manager wishing to evaluate customers' credit records in terms of how quickly they pay and how consistent they are in paying their accounts. Evaluate the following three accounts and discuss which would be the best customer. The numbers reported are the average days the accounts receivable have been outstanding for a 5 -year period.

| GORDON | 31.1 | 31.0 | 31.6 | 31.5 | 30.9 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| FENNER | 31.3 | 31.0 | 31.4 | 31.5 | 30.4 |
| BEAN | 31.0 | 30.9 | 31.5 | 31.9 | 30.8. |

Obviously all three accounts are good ones, but does this method really give us a way to distinguish them?
3. Compare and discuss the income distributions of the following two groups of technicians, one of which works abroad and one in this country.

| (A) INCOME | (B) INCOME |
| :---: | :---: |
| (DOLLARS) | (POUNDS) |
| 10,000 | 3,000 |
| 12,000 | 3,800 |
| 9,500 | 4,500 |


| 13,500 | 2,800 |
| :--- | :--- |
| 14,000 | 3,500 |
| 11,600 | 4,200 |

### 7.4. LINREG: A LINEAR REGRESSION CALCULATOR

The purpose of relationship analysis is to look at two, or more, sets of data to discover if any correlation exists between them. The usual motive for such analysis is to discover patterns in the data that might allow us to predict, or control, the outcome of some event which is represented by one set of data in terms of another. It is not a simple matter to write a computer program which accepts information, then looks for any possible existing relationship. In practice, a program is designed to test whether a certain relationship exists between sets of data. In fact, we usually can do better than this. We can compare populations to obtain a reading of how well they correlate with respect to a specific relationship.

In the case of linear regression, two sets of data are compared to see if a linear relationship exists. The measure of the quality of this relationship is called the correlation coefficient. If the correlation coefficient is 1 , the correlation is perfectly linear; if it equals 0 , no linear relationship exists at all. Any value between 0 and 1 will give an insight on how strong the relationship really is. Gaining experience in dealing with particular data sets, under conditions that you understand, is the best way to get a good grasp on what the correlation coefficient really means.

In addition to getting a reading, in a given situation, of how strong a linear relationship is, this program also allows the prediction of one variable in terms of the other. An example will make this clear. We will deal here with two variables, called $X$ and $Y$. Suppose the $X$ values are the first eight months of the year, January through August, and let the Y values be the total sales, by the month, for a particular salesperson. We, then, have two data sets with eight data points in each set:
X: X1, X2, X3, X4, X5, X6, X7, X8
and
Y: Y1; Y2, Y3, Y4, Y5, Y6, Y7, Y8.

Using LINREG, the actual values of these variables are entered on the worksheet and the sheet recalculated. Among other things, the output includes the correlation coefficient. Suppose it is .94. This indicates a strong linear relationship. On this basis, this person's sales may be predicted for the ninth month. To do this, enter 9 as an $X$ value, then compute what the corresponding Y value is (recalculate the worksheet). The result will be the estimate of September sales for the salesperson.

The discussion to this point probably raises more questions than it has answered. For one thing, what if we want to predict sales for the next full year. Is this reasonable? Probably not, but the intent here is to make short term predictions relative to the time frame defined by the data input into the program. Since 8 months of data was entered, a 12 -month prediction would not usually make much sense; however, a 2-month prediction might be used as a reasonable guide for planning purposes.

An obvious criticism of this program is it tests for a linear relationship, and few relationships are linear. Although this is true, any smooth curve can be approximated by a series of sufficiently small, straight lines. This is precisely where common sense comes into play, and why LINREG is so useful. No hard and fast rules exist. Examine the data carefully. Verify that the correlation coefficient is close to 1. Ask yourself if, intuitively, the predictions make sense. Explore other similar situations to see what happened in the past. With this approach you will find LINREG a powerful planning tool. Its applications are limited only by your imagination.

The row labeled "ESTIMATED VALUES" is used to compute a Y-value, given an X-value, or to compute an X-value, given a Y-value, based on the estimating line which best fits the data input into the program. This line is defined by a linear equation of the form:

$$
\mathrm{Y}=\mathrm{MX}+\mathrm{B} ;
$$

where $M$ is the slope of the line and $B$ is the $Y$-intercept.

### 7.5. HOW TO USE LINREG

The input for LINREG is a set of paired data that is entered in two columns on the worksheet. A typical output is displayed in Table 7-3. Note that the windows have been removed in this screen to show you the actual layout of the calculator on the worksheet. The flow diagram in Fig. 7-3 walks you through a typical problem using LINREG.


## Table 7-3

In addition to the problem illustrated in the walkthrough above, this calculator can be used to estimate a Y -value given an X -value or an X -value given a Y -value. This is done in the row labeled "ESTIMATED VALUES." For example, to compute a Y-value, move the cursor to the "X-value" cell and enter a numerical value, say 6. To enter this value

Press: 6 and RETURN.
Now, move the cursor to the "Y-value" cell and compute Y.
Press: V, Y, RETURN and !!.
An analogous calculation is made to compute an $X$ value.

## LINREG: A LINEAR REGRESSION CALCULATOR

 EXAMPLE OF INPUT AND OUTPUT PROCEDURESINITIAL CONDITIONS:
(1) LINREG loaded and ready to use
(2) Manual recalculation set
(3) Cursor at the top of X column, R17C2

PROBLEM DEFINITION: Enter the following data for linear regression computations:

| $X$ | $Y$ |
| ---: | ---: |
| 5 | 31 |
| 11 | 40 |
| 4 | 30 |
| 5 | 34 |
| 3 | 25 |
| 2 | 20 |

## ACTION ON KEYBOARD RESPONSE ON SCREEN



Fig. 7-3

## Helpful Hints on Using LINREG

A. It is sometimes necessary to clear the input columns when, for instance, you wish to enter fewer values than already are entered in the column. Fig. 7-4 illustrates how this is done.
B. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
C. You may wish to print out this particular worksheet so that you have a hardcopy for your own records. Detailed instructions on printing procedures are given in Section 4.12 which discusses how to print amortization schedules.

### 7.6. EXAMPLES USING LINREG

1. The following table shows 15 weeks of sales in thousands of dollars of a retail store in Seattle and a branch store in Bellevue.

| SEATTLE | BELLEVUE |
| :--- | :--- |
| 64 | 34 |
| 60 | 29 |
| 71 | 44 |
| 71 | 46 |
| 63 | 33 |
| 60 | 28 |
| 71 | 45 |
| 64 | 35 |
| 69 | 42 |
| 64 | 35 |
| 68 | 40 |
| 72 | 47 |
| 68 | 41 |
| 73 | 48 |
| 72 | 47 |

Is there a strong relationship between the sales in these two stores? If the Seattle store reaches $\$ 75 \mathrm{~K}$, what sales can we expect from the

Bellevue store? Plot these values on a sheet of graph paper, then draw an estimation line defined by the slope and intercept values given on the output screen. This is called a scatter diagram, and is one of the best ways to visually evaluate the strength of the relationship defined by these two sets of data.
2. The following data was gathered by a popcorn vendor during a period of six weeks. It shows the price of a bag of popcorn, and the number of bags sold each week. If his cost is $\$ .07$ per bag at what price level will he maximize his profits? Assume the relationships defined by his data accurately reflect his sales.

## CLEARING AN ENTRY COLUMN

## INITIAL CONDITIONS:

(1) One of the calculators loaded and ready to use
(2) Cursor located at the top of column to be cleared

PROBLEM DEFINITION: Clear the top six cells of an input column of existing entries with the COPY DOWN command.

ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN



Fig. 7-4

| PRICE | NUMBER SOLD |
| :--- | :--- |
| .15 | 440 |
| .20 | 430 |
| .25 | 450 |
| .30 | 370 |
| .40 | 340 |
| .50 | 300 |

This problem may be solved by one of several ways. It is not elementary, and will provide a good challenge to develop a worksheet which displays his profits at different price levels. Use your estimating equation from LINREG.

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## Chapter 8

## Real Estate Finance

### 8.1. VRM: VARIABLE RATE MORTGAGES



Fixed rate mortgages also have a fixed payment amount. A variable rate mortgage requires the monthly payments be adjusted to reflect changes in the mortgage rate applied to the loan. Rates are usually not changed more often than every six months, with a maximum rise of $0.5 \%$ a year and $2.5 \%$ over the life of the loan.

The problem, then, is to compute the monthly payments given the original loan amount, its term, the rates and the periods of time (terms) they are in effect. These worksheets are particularly useful in this respect because, usually, we do not know the rates in advance, and it is helpful to be able to forecast a best and worst case. A typical screen is displayed in Table 8-1.

### 8.2. HOW TO USE VRM: VARIABLE RATE MORTGAGES

All of the mortgage calculators have basically the same screen format and the same entry and recalculation procedures. The step-by-step instructions in Fig. 8-1 walk you through a typical problem using VRM. The corresponding screen is displayed in Table 8-1.

## Helpful Hints on Using VRM

A. Sometimes it is necessary to clear an input column when, for example, fewer values are to be entered than already are in the column. Fig. 8-2 illustrates how this is done.
B. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
C. You may wish to print out this particular worksheet so that you

## VARIABLE RATE MORTGAGE CALCULATOR

 EXAMPLE OF INPUT AND OUTPUT PROCEDURES
## INITIAL CONDITIONS:

(1) VRM loaded and ready to use
(2) Manual recalculation set
(3) Cursor at LOAN AMOUNT cell, R4C4

PROBLEM DEFINITION: Compute the payment amounts of a variable rate mortgage given the following inputs:

$$
\text { LOAN AMOUNT . . . . . . . . . . . . . . . . . . . } \$ 80,000
$$

TERM OF LOAN (MONTHS) . . . . . . . . . . 360
36 MONTHS ......................... . $14.50 \%$
24 MONTHS ......................... . $15.74 \%$
60 MONTHS ......................... . $15.50 \%$
240 MONTHS ......................... $14.00 \%$

## RESULT OR RESPONSE ON SCREEN

Enters ' 80000 ' in the LOAN AMOUNT cell and moves cursor to the next input cell.

Enters: '360' in the TERM OF LOAN cell and moves the cursor to the next input cell.

Enters: '36' in the TERM column and moves the cursor to the next input cell.


Enters: '24' in the TERM column and moves the cursor to the next input cell.

Fig. 8-1

## RESULT OR RESPONSE ON SCREEN



Fig. 8-1 - cont.
have a hardcopy for your own records. Detailed instructions on printing procedures are given in Section 4.12 which discusses how to print amortization schedules.

### 8.3. EXAMPLES USING VRM

1. Using the example previously given in the walkthrough, change the interest rates during each period to reflect a best and worst case. For instance, during the first period the rate given was $14.5 \%$, so a best case might be $14 \%$ and a worst case $15 \%$. Plug these values into VRM and compare your results.

## FINANCIAL PLANNING FOR MULTIPLAN ${ }^{\text {TM }}$ AND THE APPLE II®



## Table 8-1

2. What is the difference in the total interest paid over the life of the loan in your best and worst case in example no. 1? What conclusions, if any, can you draw from this example? What variable (payment, interest, etc.) is more sensitive to a change in interest rates in this situation?

### 8.4. GPM: GRADUATED PAYMENT MORTGAGES

Negative amortization is still a controversial subject; nevertheless, GPM's (not Gyp'em's) are available, and are often a realistic financing instrument under the right conditions. Now you can examine the alternatives for yourself.

With GPM's, lower payments in the early years of a loan are achieved through negative amortization. Thus, during this period, the principal of the loan increases because the monthly payments do not usually cover all of the interest for the month. For example, if you had an $\$ 80,000$ mortgage at $12 \%$ with monthly payments, you would be
paying $1 \%$ per month in interest. In the first month, your interest payment would be $\$ 800$. If your payment for the first month was only $\$ 600$, then the principal would increase by $\$ 200$, and the balance of the loan, after one month, would be $\$ 80,200$.

This is the basic idea and is usually implemented as described in the following example. Suppose we have an $\$ 80,000$ mortgage at $14 \%$ for a period of 30 years. A typical example of a graduated payment schedule might be:
$75 \%$ of level payments for 36 months $90 \%$ of level payments for 24 months $100 \%$ of level payments for 300 months.

## CLEARING AN ENTRY COLUMN

INITIAL CONDITIONS:
(1) One of the calculators loaded and ready to use
(2) Cursor located at the top of column to be cleared

PROBLEM DEFINITION: Clear the top six cells of an input column of existing entries with the COPY DOWN command.


Fig. 8-2

The problem is how to determine what the level payments are. If the payment amount is computed in the usual way, then only partial payments are made for five years, the full $\$ 80,000$ will not be paid, and a balloon payment will be required at the end of the loan period. The level payments must be set at exactly the amount that will pay off $\$ 80,000$ with the preceding schedule.

To solve this problem, we resort to a very common and elegant mathematical dodge. The trick is to assume the level payment is exactly $\$ 1.00$, and the amount that will pay off is calculated; i.e., the following payment stream is assumed:

$$
\begin{aligned}
& \$ .75 \text { for } 36 \text { months } \\
& \$ .90 \text { for } 24 \text { months } \\
& \$ 1.00 \text { for } 300 \text { months. }
\end{aligned}
$$

Its present value is computed, which is $\$ 75.71$. The correct question, which is "How many times do we have to pay off $\$ 75.71$ in order to pay off $\$ 80,000 ?^{\prime \prime}$, then, is asked. The answer, of course, is:

$$
\$ 80,000 / \$ 75.71=\$ 1056.65
$$

Therefore, our level payment is $\$ 1056.65$, and our payment schedule becomes:

$$
\begin{aligned}
75 \% \text { of } 1056.65 & =792.49 \text { for } 36 \text { months } \\
90 \% \text { of } 1056.65 & =950.99 \text { for } 24 \text { months } \\
100 \% \text { of } 1056.65 & =1056.65 \text { for } 300 \text { months } .
\end{aligned}
$$

### 8.5. HOW TO USE GPM: GRADUATED PAYMENT MORTGAGES

All of the mortgage calculators have basically the same screen format and the same entry and recalculation procedures. The step-by-step instructions in Fig. 8-3 walk you through a typical problem using GPM. The corresponding screen is displayed in Table 8-2.

## Helpful Hints on Using GPM

A. It is sometimes necessary to clear an input column when, for instance, you wish to enter fewer values than already are entered in the column. Fig. 8-4 illustrates how this is done.

## GRADUATED PAYMENT MORTGAGE CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) GPM loaded and ready to use
(2) Manual recalculation set
(3) Cursor at LOAN AMOUNT cell, R4C4

PROBLEM DEFINITION: Compute the payment amounts of a graduated payment mortgage given the following inputs:

LOAN AMOUNT . . . . . . . . . . . . . . . . . . $\$ 80,000$
ANNUAL INTEREST RATE . . . . . . . . . . . 14.00\%
TERM OF LOAN (MONTHS) . . . . . . . . . 360
36 MONTHS ........................ . $75 \%$

24 MONTHS ......................... . $90 \%$
300 MONTHS ........................... 100\%

## RESULT OR RESPONSE ON SCREEN



Fig. 8-3


Fig. 8-3 - cont.
B. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
C. You may wish to print out this particular worksheet so that you have a hardcopy for your own records. Detailed instructions on printing procedures are given in Section 4.12 which discusses how to print amortization schedules.

### 8.6. EXAMPLES USING GPM

1. You wish to purchase a $\$ 95,000$ home with a $\$ 10,000$ down payment. The maximum payment you can make for the next three years is $\$ 800$ a month, at which time you expect a salary increase which will permit more generous payments. Current interest rates are at $6 \%$ to $17.5 \%$. Can you structure a GPM that will reasonably meet these requirements?


## Table 8-2

2. Assuming a GPM payment schedule of $75 \%, 90 \%$, and $100 \%$, such as suggested in the first example above, determine the maximum amount you could borrow at $16 \%$ for 30 years, if your payment ceilings in these three periods are $\$ 800, \$ 900$, and $\$ 1,200$, respectively.

### 8.7. GPAM: GRADUATED PAYMENT ADJUSTABLE MORTGAGES

The GPAM is a graduated payment mortgage with a roll-over option every 3 to 5 years. Thus, the borrower may have reduced payments in the early years of the loan, and the lender may adjust interest rates at the agreed on roll-over dates. The GPAM is, simply, a combination of the concepts behind variable rate mortgages and graduated rate mortgages. In situations where the mortgage rate is subject to renegotiation, the GPAM offers more flexibility than either of these other two mortgage options.

Typical inputs might be the following. An $\$ 80,000$ loan for 30 years with graduated payments set up as percentages using the following associated interest rates:
Ist loan . . . . . . . . . . . . . . . . . . . . . . $75 \%$ for 36 months at $14.0 \%$
2nd loan . . . . . . . . . . . . . . . . . . . . . $100 \%$ for 24 months at 60 months at $15.5 \%$
3rd loan . . . . . . . . . . . . . . . . . $100 \%$ for 60 months at $14.0 \%$
nth loan . . . . . . . . .
e, however, future interest rates are not usually known in ad-
ce. A contract will usually specify upper limits, so use these figures
an analysis of the worst case.

## CLEARING AN ENTRY COLUMN

## INITIAL CONDITIONS:

(1) One of the calculators loaded and ready to use
(2) Cursor located at the top of column to be cleared

PROBLEM DEFINITION: Clear the top six cells of an input column of existing entries with the COPY DOWN command.

## RESULT OR RESPONSE ON SCREEN

## ACTION ON KEYBOARD



Fig. 8-4

### 8.8. HOW TO USE GPAM: GRADUATED PAYMENT ADJUSTABLE MORTGAGES

All of the mortgage calculators have basically the same screen format and the same entry and recalculation procedures. The step-by-step instructions in Fig. 8-5 walk you through a typical problem using GPAM. The corresponding screen is displayed in Table 8-3.

## GRADUATED PAYMENT ADJUSTABLE MORTGAGE CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

INITIAL CONDITIONS:
(1) GPAM loaded and ready to use
(2) Manual recalculation set
(3) Cursor at LOAN AMOUNT call, R4C5

PROBLEM DEFINITION: Compute the payment amounts of a graduated payment mortgage given the following inputs:

LOAN AMOUNT . ..................... $\$ 80,000$
TERM OF LOAN (MONTHS) . . . . . . . . . 360
36 MONTHS . . . . . . . 14.00\% . . . . . . . . 75\%

24 MONTHS . . . . . . . 15.50\% . . . . . . . . $90 \%$
60 MONTHS . . . . . . $15.60 \%$. . . . . . . . 100\%
240 MONTH ........14.05\% ........ 100\%

## RESULT OR RESPONSE ON SCREEN

Enters '80000' in the LOAN AMOUNT cell and moves cursor to the next input cell.

Enters ' 360 ' in the TERM OF LOAN cell and moves the cursor to the next input cell.

Enters ' 36 ' in the TERM column and moves the cursor to the next input cell.

Enters ' $14 \%$ ' in the RATE column and moves the cursor to the next input cell.

Enters '75\%' in the \%/PMT column and moves the cursor to the next input cell.

Fig. 8-5

## RESULT OR RESPONSE ON SCREEN

ACTION ON KEYBOARD

> Enters '24' in the TERM column and moves the cursor to the next input cell.

Enters ' $15.5 \%$ ' in the RATE column and moves the cursor to the next input cell.

Enters ' $90 \%$ ' in the \%/PMT column and moves the cursor to the next input cell.

Enters ' 60 ' in the TERM column and moves the cursor to the next input cell.

Enters ' $15.6 \%$ ' in the RATE column and moves the cursor to the next input cell.

Enters ' $100 \%$ ' in the \%/PMT column and moves the cursor to the next input column.

Enters: ' 240 ' in the TERM column and moves the cursor to the next input cell.

Enters '14.05\%' in the RATE column and moves the cursor to the next input cell.

Enters ' $100 \%$ ' in the
Press: 1, 0, 0, \%, \%/PMT column and recalculates the worksheet. Displays payment amounts in the PAYMENT column.

Fig. 8-5 - cont.

## Helpful Hints on Using GPAM

A. It is sometimes necessary to clear an input column when, for instance, you wish to enter fewer values than already are entered in the column. Fig. 8-6 illustrates how this is done.
B. Should you accidentally destroy formulas on a calculator by writing over them, simply reload the calculator from your master diskette.
C. You may wish to print out this particular worksheet so that you have a hardcopy for your own records. Detailed instructions on printing procedures are given in Section 4.12 which discusses how to print amortization schedules.


## Table 8-3

### 8.9. EXAMPLES USING GPAM

1. Suppose mortgage rates are at $18.25 \%$. You expect them to decline at an average rate of $.3 \%$ per year for the next five years, then bottom
out at $15 \%$ for the long run. Using our pattern in the above example, what can you expect for payments on a 30 -year, $\$ 80,000$ loan?
2. On a 30 -year loan, structured as in example no. 1 , what is the maximum loan you can afford if your budget can handle, at most, $\$ 800$ for the next 3 years, $\$ 900$ for the 2 years following that, and $\$ 1,200$ thereafter?

### 8.10. WRAP: WRAPAROUND MORTGAGES

A wraparound mortgage is a common form of secondary financing in which the face value of the second (wraparound) loan is equal to the balance of the first loan plus the amount of the new financing. Significant advantages to both the borrower and the lender are found in a wrap.

It works like this. You need some money, $\$ 100,000$. Suppose you hold a property subject to an $11.75 \%$ first mortgage with 12 years of $\$ 2,000 /$ month payments remaining. Current market mortgage rates are at $15 \%$, or higher, and the first mortgagee cannot, or will not, refinance the mortgage and increase the loan amount. Prepayment of the first mortgage may be barred or subject to heavy penalties.

Since the first mortgage is at a desirable interest rate, you go to a new lender to negotiate a wraparound loan that works like this:
A. The new lender advances you $\$ 100,000$ in cash.
B. You execute a promissory note in his favor in the amount of $\$ 100,000$ plus the balance on your first mortgage, which is $\$ 154,044.57$. The face value of the new loan is, then, $\$ 254,044.57$.
C. The interest rate on your new loan is not $15 \%$, but $14 \%$, at least one full percentage point below prevailing interest rates.
D. You pay the new lender debt service on the new loan and he/she pays the debt service on the first mortgage.
E. The new lender ends up receiving a return of $15.9 \%$ on the
$\$ 100,000$ advanced to you, almost one full percentage point above prevailing mortgage rates. This upside leverage results because the new lender is receiving $14 \%$ on the $\$ 100,000$ actually advanced, plus the $2.25 \%$ difference between the $14 \%$ you are now paying on the old money and the $11.75 \%$ that must be paid to the old lender.

This entire transaction is summarized in Table 8-4.

### 8.11. HOW TO USE WRAP: A WRAPAROUND MORTGAGE CALCULATOR

The input for WRAP is the data to be entered in the first nine cells of the screen reproduced in Table 8-4, four items on the FIRST MORTGAGE and four items on the WRAPAROUND MORTGAGE. The

## CLEARING AN ENTRY COLUMN

## INITIAL CONDITIONS:

(1) One of the calculators loaded and ready to use
(2) Cursor located at the top of column to be cleared

PROBLEM DEFINITION: Clear the top six cells of an input column of existing entries with the COPY DOWN command.


Fig. 8-6
output consists of the 14 items listed under SUMMARY. The step-bystep instructions in Fig. 8-7 walk you through a typical problem using GPAM. The corresponding screen is displayed in Table 8-4.


## Table 8-4

## Helpful Hints on Using WRAP

A. Should you accidentally destroy formulas on a calculator by writing over them, simply load the calculator from your master diskette.
B. You may wish to print out this particular worksheet so that you have a hardcopy for your own records. Detailed instructions on printing procedures are given inSection 4.12 which discusses how to print amortization schedules.

### 8.12. EXAMPLES USING WRAP

1. We are given a $9.5 \%$ first mortgage with $\$ 650$ monthly payments

## WRAPAROUND MORTGAGE CALCULATOR

## EXAMPLE OF INPUT AND OUTPUT PROCEDURES

## INITIAL CONDITIONS:

(1) WRAP loaded and ready to use
(2) Manual recalculation set
(3) Cursor at LOAN AMOUNT cell, R5C4


## ACTION ON KEYBOARD

## RESULT OR RESPONSE ON SCREEN

Enters: ' 144 ' in the REMAINING TERM OF LOAN cell and moves cursor to the next input cell.

Enters: '11.75\%' in the ANNUAL INTEREST RATE cell and moves the cursor to the next input cell.

Enters: '60' in the TERM column and moves the cursor to the next input cell.

Fig. 8-7


Fig. 8-7 - cont.
with 16 years remaining to maturity. If you advance $\$ 25,000$ on a wrap at $14 \%$ for 20 years, what is your yield?
2. You have a standing loan of $\$ 40,000$ due in eight years, the interest rate is $12 \%$ and no periodic payments are required. You want another $\$ 20,000$ for 10 years and have found a new lender who will wrap this for you if he can achieve a minimum yield of $16.5 \%$. Into what interest rate will this put you, and what will be your monthly payment?

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0

## Glossary

## Accelerated depreciation

Any method of depreciation where the depreciation amounts decrease progressively each period. This gives a tax advantage in the early life of an asset.

## Accumulated interest

The accumulated interest on a cash flow is the difference between the cash value of the cash flow (the sum of the payments) and its present value. It is the accrued interest on the interest bearing principal.

## Amortization

The process of paying off a debt over a fixed period of time.

## Annuity

A periodic cash flow with level payments. A mortgage or lease payments are typical examples.
Annuity due
An annuity with payments made at the beginning of each period.

## Balloon payment

A payment made at the end of an annuity which is larger than the preceding payments.

## Book value

As used here, the difference between the acquisition cost and the depreciation taken on an asset.

## Break-even analysis

The analysis of the relationship between fixed costs, variable costs, and profits.

## Cash flow

A schedule of payments. The cash value of a cash flow is the sum of all the payments. Cash inflows are positive and cash outflows are negative.

## Coefficient of variation

A relative measure of dispersion, the standard deviation divided by the mean.

## Compound growth

In finance, growth which is proportional to the principal plus the accumulated interest. Interest is compounded periodically as opposed to simple growth in which interest is proportional to the principal amount only.

## Compounding frequency

Interest rates are reported on an annual basis, but they are not always compounded annually. The compounding frequency is the number of times per year interest is compounded.

## Continuous growth

The limit of discrete growth as the compounding frequency approaches zero. If interest is compounded continuously per period at a rate P , then the effective interest rate per period is $\exp (\mathrm{P})-1$, where P is expressed as a decimal.

## Continuous compounding

Same as continuous growth.

## Contribution margin

The gross profit margin as a percent of the unit selling price. It is computed with the formula:

> (unit selling price - unit cost)/(unit selling price).

## Correlation coefficient

Measures the closeness of relationship between two variables. One $(1)$ is a perfect correlation, zero ( 0 ) is no correlation.

## Depreciation

The process of allocating the cost of an asset to the periods of benefit for tax purposes.

## Discount rate

The interest rate which is used to compute a present value from a stream of future values.

## Discounted cash flow analysis

Refers to any method used to measure a cash flow by discounting future values. Used for ranking investment proposals. A yield is obtained if a present value is equated to a stream of future values by a discount rate. The internal rate of return and the financial management rate of return are examples of this technique. If a discount rate is given, then usually a net present value is computed.

## Double declining balance

A method of depreciation that uses a constant percentage rate to compute the period depreciation from the book value.

## Earned interest

The accumulated interest on a cash flow is the difference between the cash value of the cash flow (the sum of the payments) and its present value. It is the accrued interest on the interest bearing principal.

## Effective yield

The annual interest rate which equates a present value with a future value. Use the COMP calculator to compute the effective yield. For instance, if $\$ 45.00$ grows to $\$ 56.00$ in two years the effective yield is $11.55 \%$. Recall that F is one, N must be set to two, and either the present value or future value is entered as negative.

## Financial management rate of return

The interest rate which equates the present value of the cash outflows of a cash flow discounted at an appropriate rate to the future value of the cash inflows of the cash flow compounded at an appropriate reinvestment rate.

## Fixed costs

A cost that does not vary with the volume of activity of a business during a specified period of operation.

## Fixed rate mortgage

An ordinary annuity in which the payments are viewed as amortizing a debt equal to the present value. Contrast with variable rate mortgage.

## Frequency of compounding

See compounding frequency.

## Future value

A present value which is compounded at a given rate and period of time to a future value.

## Graduated payment adjustable mortgage

The GPAM is a graduated payment mortgage with a roll-over option every three to five years. Thus the borrower may have reduced payments in the early years of the loan and the lender may adjust interest rates at the agreed on roll-over dates.

## Graduated payment mortgage

Lower payments in the early years of the loan are achieved through negative amortization. Thus, during this period, the principal of the loan is increasing.

## Growth rate

Same as nominal interest rate.

## Initial investments

In a cash flow, the cash value associated with period zero is regarded as an initial investment. It is an outflow and, therefore, negative.

## Internal rate of return

The internal rate of return of a cash flow is the interest rate which equates the initial investment with the present value of the remaining payment stream. Equivalently, it is the interest rate that makes the net present value zero.

## Linear regression

An analytic technique that derives an equation for a line that statistically is the best representation of the given data. The object is to predict the value of one variable in terms of another.

## Margin

Generally in accounting a margin, or profit margin, is revenues less expenses. It may be expressed as a percent of revenues. See contribution margin.

## Mean

The arithmetic average of a set of numerical data.

## Mortgage

A legal claim on an asset given to a lender by a borrower in return for a loan. Most mortgage loans are ordinary annuities.

## Net present value

The difference between the present value of a stream of payments discounted at a given interest rate and the initial investment associated with the payments.

## Newton-Raphson Method

A method of approximating the value of a function by successive approximations each of which is more accurate than the last.

## Nominal interest rate

An interest rate stated as an annual rate.

## Normal distribution

A symmetrical, bell-shaped probability distribution that measures how data is distributed around the mean.

## Number of compounding periods

Used with compound interest and annuities. It is the total number of compounding periods to maturity.

## Ordinary annuity

An annuity with payments made at the end of each period.

## Payment

In a cash flow, an individual cash value regarded as an outflow on one side of the transaction and an inflow on the other.

## Population standard deviation

The standard deviation of a total population.

## Present value

A measure of the value of a cash flow at its beginning date. It is computed by discounting all future values by an appropriate discount rate.

## Regression

See linear regression.

## Salvage value

The value of an asset at the end of its useful life.

## Sample standard deviation

The standard deviation of a sample set of data from a total population. Because this statistic is used for estimation based on a sample, it is larger than the population standard deviation by a factor equal to the square root of $N /(N-1)$, where $N$ is the sample size.

## Selling price

The selling price is used in break-even analysis in conjunction with the unit cost, contribution margin, and fixed cost to compute the level of sales in terms of dollar sales and units sold at which the firm will just meet its cost of operations.

## Simple interest

Interest calculated on the present value or principal of a loan. The formula is

$$
\text { interest }=\text { principal } \times \text { interest rate } \times \text { time } .
$$

## Sinking fund

An annuity due which is regarded as a payment stream accumulating to a future value.

## Standard deviation

A statistical measure of the variability of a set of data from the mean. The greater the standard deviation, the greater the average deviation of a data point from the mean.

## Straight-line depreciation

A method of depreciation that results in level depreciation charges throughout the life of an asset.

## Sum-of-the-years-digits depreciation

An accelerated method of depreciation where the depreciation in each period is a fraction of the total depreciable value of an asset.

## Unit cost

The unit cost is used in break-even analysis in conjunction with the selling price, contribution margin, and fixed cost to compute the level of sales in terms of dollar sales and units sold at which the firm will just meet its cost of operations.

## Useful life

In depreciation calculations the useful life is the period of time over which a firm is allowed to depreciate an asset.

## Variable costs

Variable costs are costs that change with the volume of activity of a firm.

## Variable rate mortgage

A mortgage in which the interest rate and, therefore, the payments may vary according to a predetermined time schedule, usually in 2-, 3 -, or 5 -year increments. The interest rate is usually tied to some index and definite limits of its variability are set over the life of the loan.

## Variance

A statistical measure of the variability of a set of data from the mean. It is equal to the square of the standard deviation.

## Wraparound mortgage

A method of financing in which a second loan (the wraparound mortgage) is taken out with a face value equal to the balance of the first loan plus the amount of new funds advanced.

## Yield

The rate of return on an investment. The internal rate of return and the financial management rate of return are examples of two methods of computing a yield.

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