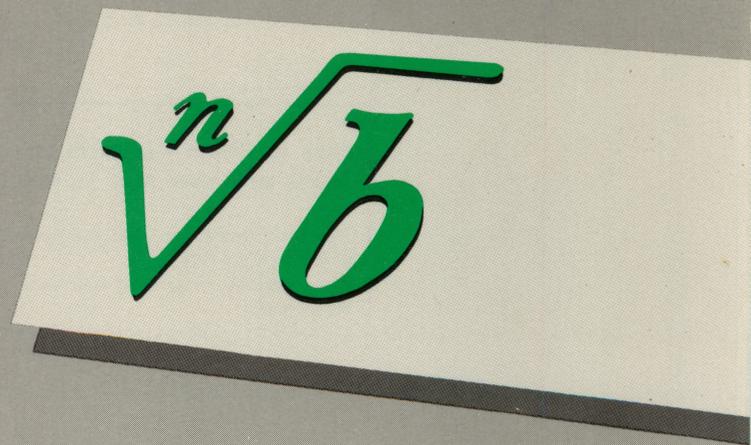# Using Algebra Plotter Plus

Includes: User's Manual · Worked Examples · Scripted Demonstrations
Enrichment Topics · 21 Activity Sheets · Answers · Correlation to
*Algebra, Structure and Method, Books 1 and 2*

# Algebra

## Structure and Method
### Books 1 and 2

$$\frac{1}{b}^{-3}$$

$$4y$$

$$\sqrt[n]{b}$$

$$f(x)$$

Houghton Mifflin

# Using Algebra Plotter Plus

# Algebra

**Brown**
**Dolciani**
**Sorgenfrey**
**Cole**
**Kane**

*Structure and Method*
*Books 1 and 2*

Includes:
- User's Manual
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- Enrichment Topics
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- Answers
- Correlation to *Algebra, Structure and Method, Books 1 and 2*

**Houghton Mifflin Company · BOSTON**

Atlanta · Dallas · Geneva, Illinois · Palo Alto · Princeton · Toronto

## About the Disk Designer and Programmer

**David L. Myers** is the designer and Apple programmer of the
*Algebra Plotter Plus* disk, and the writer of many of the algebra
activities in this booklet. He is Computer Coordinator and
Mathematics Teacher at the Winsor School, Boston, MA.

Mr. Myers is the senior author of Houghton Mifflin's
*Computer Programming in BASIC* textbook.

# Contents

# Before You Begin

Follow the instruction sheet included with your *Algebra Plotter Plus* disk. Make
a copy of the disk before you start using it.

   Insert the disk into a drive and turn your computer and monitor on. Make sure
your keyboard is attached securely to your computer. The Apple disk will boot
automatically. For the IBM disk, you will have to load DOS first, or follow the
directions on the instruction sheet to create a self-booting disk.

# Overview of the Disk

The *Algebra Plotter Plus* disk can be used independently by students or by the teacher as a classroom presentation device.

## What is the *Algebra Plotter Plus* software?

The *Algebra Plotter Plus* software is intended to help students understand the graphic representation of functions, conics, and statistical data.

## What can *Algebra Plotter Plus* do?

Plotters: These programs allow the user to enter up to three equations at once, decide the scales of the axes, and plot the graphs. The following programs are included on the main menu: **Line Plotter, Parabola Plotter, Absolute Value Plotter, Function Plotter, Inequality Plotter,** and **Conics Plotter.**

Quizzes: These programs check understanding by presenting graphs of a given type, and asking the student to determine the equation associated with the graph. The following programs are included on the main menu: **Line Quiz, Parabola Quiz, Conics Quiz,** and **Circular Function Quiz.**

Utilities: **Matrix Reducer**: a utility program that allows students to reduce matrices by specifying row operations;
**Statistics Spreadsheet**: a utility program for presenting histograms, scatterplots, and regression lines of spreadsheet data;
**Sampling Experiment**: a simulation that chooses samples from a population with specified statistical characteristics.

## How can I learn to use the software?

Refer to the User's Manual and the Worked Examples (pages 4–15) for detailed instruction on all the options available with the *Algebra Plotter Plus* disk. You may reproduce these pages for students to use as a reference.

The Scripted Demonstrations (pp. 18–30) may be used to present and motivate lesson material, and to introduce students to the *Algebra Plotter Plus* disk. The Enrichment Topics (pp. 31–42) are provided for teachers who wish to use *Algebra Plotter Plus* more frequently. They include suggestions of how students might use the disk to make further explorations and discoveries. The Activities (pp. 43–66) are worksheets designed to reinforce specific lessons in the textbook. Students may complete these independently or in small groups.

The Correlation (pp. 16–17) charts the Scripted Demonstrations, Enrichment Topics, and Activities in the order in which they would be presented in a typical syllabus. The chart is organized on a lesson-by-lesson basis. Page references refer to this booklet.

# Algebra Plotter Plus User's Manual

Follow the instruction sheet that comes with the disk. Once you have booted the disk, you will see the title screen. Press the <RETURN> key (on Apple computers) or the <ENTER> key (on IBM computers) to get to the main menu, which lists the fourteen options shown at the right.

Use the arrow keys to move up and down this list, then select the highlighted item by pressing <RETURN> or <ENTER>.

After you select one of the programs from the main menu, it will load and you will see a program menu at the bottom of the screen. The options of a program menu are arranged in the order that you would generally use them. For example, a typical plotter program has the program menu listed below.

**Line Plotter**
**Line Quiz**
**Parabola Plotter**
**Parabola Quiz**
**Absolute Value Plotter**
**Function Plotter**
**Inequality Plotter**
**Conics Plotter**
**Conics Quiz**
**Circular Function Quiz**
**Matrix Reducer**
**Statistics Spreadsheet**
**Sampling Experiment**
**Quit**

## HELP   EQUATIONS   DRAW   SCALE   ZOOM   TABLE   OTHER   QUIT

Beginners may wish to use the HELP screen information. You must enter your EQUATIONS before you can DRAW the graphs. Likewise, you must DRAW your graphs before you can ZOOM in on them or list a TABLE of values. You may select the SCALE at any time. The OTHER option allows you to specify the grids, labels, and graph styles. Select QUIT to return to the main menu.

### Arrow keys

In any menu, one of the menu options will be highlighted. In a vertical list, press the up and down arrow keys to move the highlighting to the option you want. In a horizontal list, press the left and right arrow keys. The arrow keys are also used to move from one set of parentheses to the next when filling in numbers in an equation format. Some sample formats are shown below:

Line Plotter:          $y = (\ )x + (\ )$                or $(\ )x + (\ )y = (\ )$
Parabola Plotter:      $y - (\ ) = (\ )(x - (\ ))\wedge 2$   or $y = (\ )x\wedge 2 + (\ )x + (\ )$
Conics Plotter:        $(\ )(x - (\ ))\wedge 2 + (\ )(y - (\ ))\wedge 2 = (\ )$
                       or $(\ )x\wedge 2 + (\ )y\wedge 2 + (\ )x + (\ )y = (\ )$

Similarly, the arrow keys may be used to enter the elements of a matrix in the Matrix Reducer program, and the elements of a data set in Statistics Spreadsheet.

### <RETURN> (Apple) or <ENTER> (IBM)

When making a choice from a menu, press this key to select the currently highlighted option. Also, if a program has paused to let you read the screen, press this key to resume program execution.

### <ESC> (Escape)

Press the <ESC> key to leave an option and return to the previous menu. For example, after entering your EQUATIONS in the Line Plotter program, you may press <ESC> to go back to the Line Plotter program menu. You could then select the DRAW option to graph your equations.

---

# Worked Examples

To familiarize yourself with *Algebra Plotter Plus,* study the Worked Examples. These are step-by-step introductions to each of the programs on the disk.

In the paragraphs below and in the Activities that follow, the symbol <↵> is sometimes used to indicate pressing <RETURN> or <ENTER>. Also, the term "Select" is used to suggest highlighting a desired function and pressing <RETURN> or <ENTER>.

---

## Using the Plotters

### Line Plotter

*Use the Line Plotter program to enter linear equations in slope-intercept form and standard form. Use ZOOM to approximate solutions to systems of linear equations.*

Assume you want to find the point of intersection of the lines with equations $y = -2x + 1$ and $3x + -4y = 12$.

First you must input the equations.
Select Line Plotter from the main menu.
Select EQUATIONS. Fill in the blank parentheses so line #1 reads:

#1: y = (−2)x + (1)

Use the down-arrow key to move to line #2. Then use the right-arrow key to move to and fill in the parentheses in the standard form at the right of the screen:

#2: (3)x + (−4)y = (12)

Now present the linear equations in graphed form. Press <ESC>. Select DRAW.

The point of intersection may be estimated by inspecting the graph. A good first approximation would be (1.5, −2.0). To achieve a better approximation, change the boundaries of the graphed region by using the ZOOM feature.

Select ZOOM. Use the arrow keys to move the cross hairs over the intersection. Press ↵. Use the arrow keys to size the box as small as possible. Press ↵.

The new approximation might be (1.45, −1.91). If you like, you can zoom in again to get a better approximation. The exact algebraic solution is $(1\frac{5}{11}, -1\frac{10}{11})$, or $(1.4545\ldots, -1.9090\ldots)$.
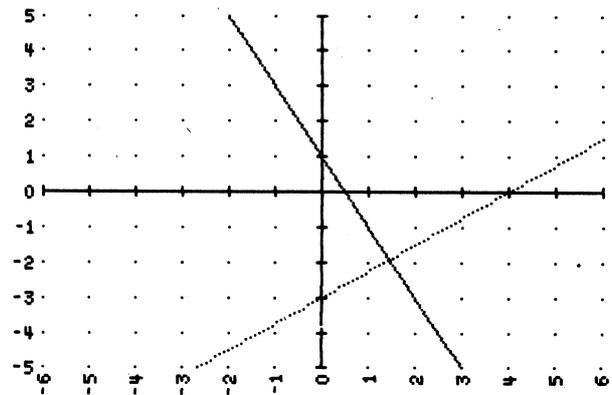


(solid) y=(-2 )x+(1 )
(dotted) (3 )x+(-4 )y=(12 )
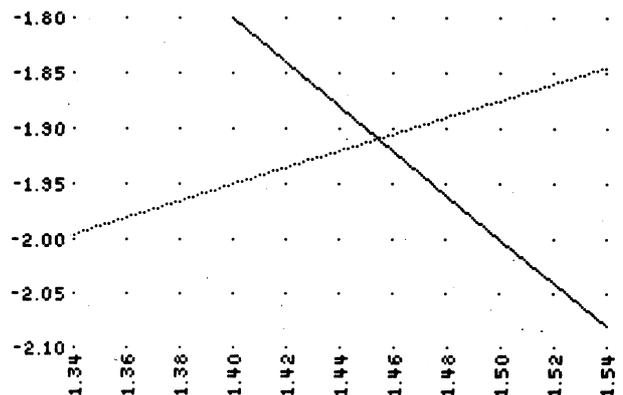


(solid) y=(-2 )x+(1 )
(dotted) (3 )x+(-4 )y=(12 )

---

## Parabola Plotter

*Use the Parabola Plotter program to explore the vertices of parabolas.*

Although Function Plotter can be used to graph any quadratic function, the Parabola Plotter program uses a format that may help students predict vertices by translation from the origin.

Select Parabola Plotter from the main menu. Select EQUATIONS. Enter the following equations:

#1: $y - (0) = (3)(x - (5))^2$

#2: $y - (2) = (4)(x - (-3))^2$

#3: $y - (-1) = (-1/5)(x - (-2))^2$

Press ‹ESC›. Select DRAW.

The first parabola has a vertex at (5, 0), the second parabola has a vertex at (–3, 2), and the third parabola has a vertex at (–2, –1). Note that large coefficients (such as 3 or 4) result in "narrow" parabolas, while a coefficient near zero (such as –1/5) results in a "wide" parabola.
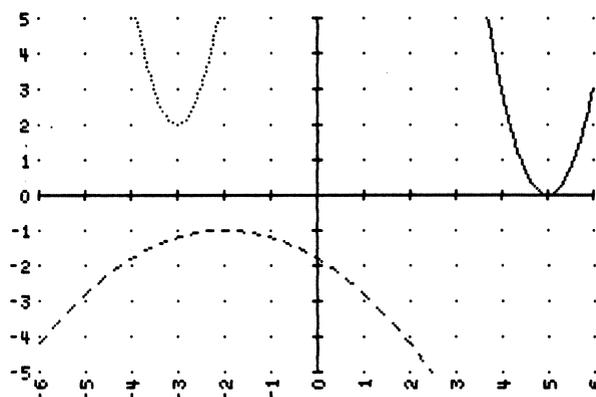
```
(solid)  y-(0 )=(3 )(x-(5 ))^2
(dotted) y-(2 )=(4 )(x-(-3 ))^2
(dashed) y-(-1 )=(-1/5 )(x-(-2 ))^2
```

## Absolute Value Plotter

*Use the Absolute Value Plotter program to explore the appearance and vertices of absolute value functions.*

Select Absolute Value Plotter from the main menu.

Enter the absolute value functions $y = |x|$, $y = |x - 2|$, and $y + 4 = 2|x + 3|$.

Select EQUATIONS. Enter the equations shown:

#1: $y - (0) = (1)\text{Abs}(x - (0))$

#2: $y - (0) = (1)\text{Abs}(x - (2))$

#3: $y - (-4) = (2)\text{Abs}(x - (-3))$

Press ‹ESC›. Select DRAW.

The program draws three V-shaped graphs.

To minimize distortion of the grid, you may want to change the SCALE.

Select SCALE. Set the following boundaries:

Least x-value: –6
Greatest x-value: 6
Least y-value: –4
Greatest y-value: 4

Press ‹ESC›. Select DRAW.
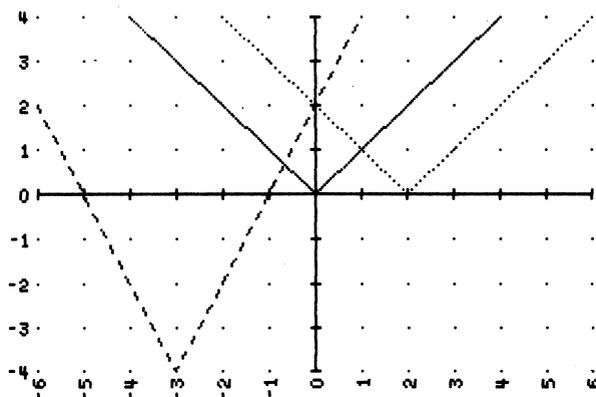
```
(solid)  y-(0 )=(1 )Abs(x-(0 ))
(dotted) y-(0 )=(1 )Abs(x-(2 ))
(dashed) y-(-4 )=(2 )Abs(x-(-3 ))
```

## Function Plotter

**Example 1:** *Use the Function Plotter program to explore polynomial functions, inverse variation, and inverse functions.*

Select Function Plotter from the main menu.
Enter three functions: the polynomial function $y = (x + 4)(x + 2)(x + 1)$, the inverse variation function $y = \frac{4}{x}$, and the function $y = \sqrt[3]{2x}$ (the inverse function of $y = \frac{1}{2}x^3$).

Select EQUATIONS. Enter the equations shown:

#1: y = (x + 4)(x + 2)(x + 1)

#2: y = 4/x

#3: y = (2x)^(1/3)

Press <ESC>. Select DRAW.

The solid graph is the cubic polynomial, which has zeros at $-4$, $-2$, and $-1$. The dotted graph is a hyperbola with the $x$- and $y$-axes as asymptotes. The dashed graph is the inverse of a cubic function.
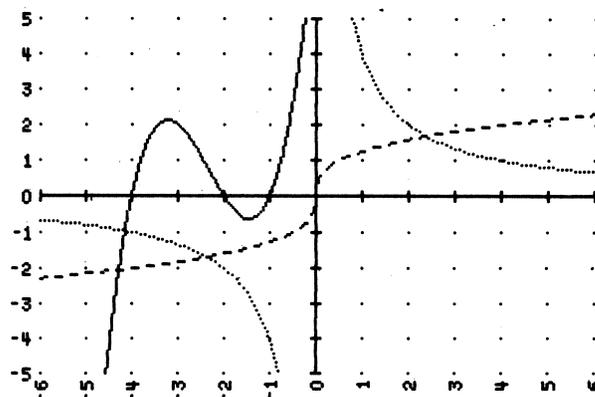


```
(solid)  y = (x+4)(x+2)(x+1)
(dotted) y = 4/x
(dashed) y = (2x)^(1/3)
```

**Example 2:** *Use the Function Plotter program to explore periods of sinusoidal functions. Also, verify a common trigonometric identity.*

The Function Plotter program allows you to plot sinusoidal functions. It is often convenient to mark the $x$-axis in multiples of $\pi$. To do so, change the scale of the $x$-axis as follows.

Select Function Plotter from the main menu. Select SCALE. Highlight "Multiples of pi" and use the right-arrow key to indicate "Yes." Press <ESC>.

Enter the following three sinusoidal functions: $y = 3\sin(x)$, $y = 3\sin(2x)$, and $y = 3\sin(4x)$.

Select EQUATIONS. Enter the equations:

#1: y = 3sin(x)

#2: y = 3sin(2x)

#3: y = 3sin(4x)

Press <ESC>. Select DRAW.



```
(solid)  y = 3Sin(x)
(dotted) y = 3Sin(2x)
(dashed) y = 3Sin(4x)
```

Notice that the period of the first function is $2\pi$, the period of the second function is $\pi$, and the period of the third function is $\frac{\pi}{2}$. In general, the period of the function $y = a\sin(bx)$ is $\frac{2\pi}{b}$.

---
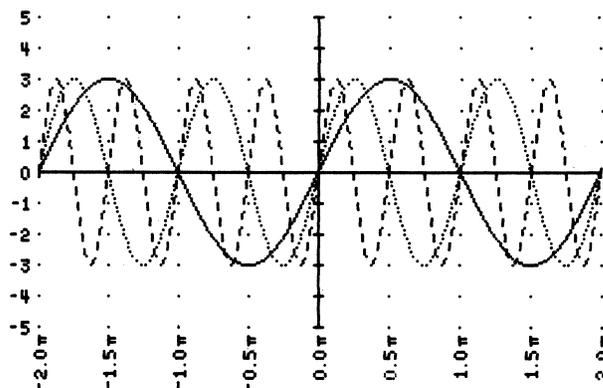
Another interesting demonstration using sinusoids involves plotting the following three equations:

#1: y = sin(x)^2

#2: y = cos(x)^2

#3: y = sin(x)^2 + cos(x)^2

Because the graphs of the first two equations involve squaring, the curves have positive y-values throughout.

　　Students will see that the first curve (solid) is a mirror image of the second curve (dotted), and that the sum of the two curves (dashed) is 1 for every value of $x$. This is a way to demonstrate the trigonometric identity: $\sin^2(x) + \cos^2(x) = 1$.
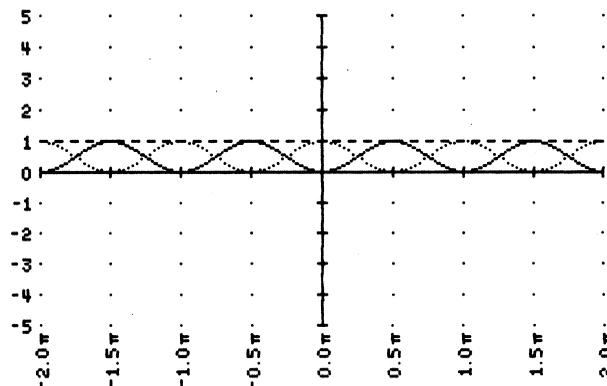
```
(solid)  y = Sin(x)^2
(dotted) y = Cos(x)^2
(dashed) y = Sin(x)^2+Cos(x)^2
```

*Example 3: Use the Function Plotter program to view exponential growth and decay curves.*

The Function Plotter can graph exponential functions of the form $y = ab^x$. When investigating real-life growth or decay models, only positive time-values ($x \geq 0$) are considered, so change the scale as shown.

Select Function Plotter from the main menu.

Select SCALE. Set the following boundaries:

Least x-value: 0
Greatest x-value: 16
Least y-value: 0
Greatest y-value: 10
Press ‹ESC›.

Enter the following exponential functions:
$y = 8(0.75)^x$, $y = 6(0.95)^x$, and $y = 2(1.2)^x$.

Select EQUATIONS. Enter the functions:

#1: y = 8(0.75)^x

#2: y = 6(0.95)^x

#3: y = 2(1.2)^x

Press ‹ESC›. Select DRAW.

The coefficients 8, 6, and 2 correspond to the three initial amounts (the y-intercepts). The three base numbers 0.75, 0.95, and 1.2 determine whether the graphs are decreasing or increasing.

　　If you use ZOOM several times, you will find that the graphs of the first two equations intersect at approximately $x = 1.217, y = 5.637$.
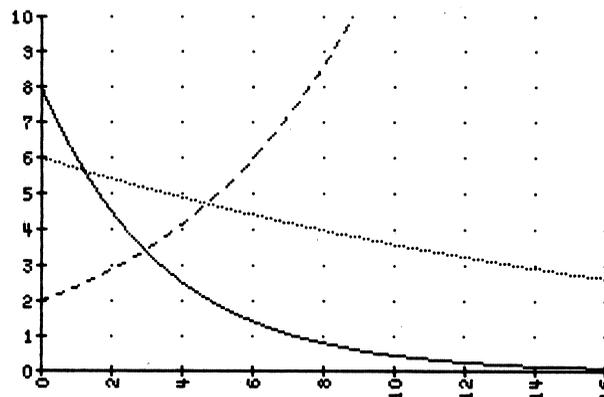
```
(solid)  y = 8(0.75)^x
(dotted) y = 6(0.95)^x
(dashed) y = 2(1.2)^x
```
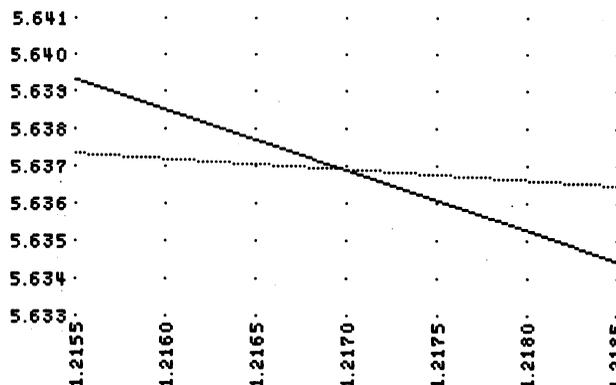
You can get a table of function values by selecting TABLE.

```
TABLE OF VALUES:  #1: y = 8(0.75)^x
                  #2: y = 6(0.95)^x
                  #3: y = 2(1.2)^x

       x     |    #1: y    |    #2: y
-------------------------------------------------
   1.2155        5.63932278      5.63734083
   1.2160        5.63851167      5.63719626
   1.2165        5.63770067      5.63705168
   1.2170        5.6368898       5.63690711

   1.2175        5.63607905      5.63676255
   1.2180        5.6352684       5.63661798
   1.2185        5.63445788      5.63647343
```

## Inequality Plotter

*Use the Inequality Plotter program to set up a linear programming problem given five restrictions.*

Assume that the restrictions that define the feasible region for a linear programming problem are as follows:

$x \geq 0$

$y \geq 0$

$y \leq 1x + 5$

$y \leq -0.5x + 9.5$

$y \leq -2x + 20$

The first two inequalities indicate that the feasible region lies entirely in the first quadrant. Set the scale to reflect this.

Select Inequality Plotter from the main menu. Select SCALE. Set the following boundaries:

Least x-value: 0
Greatest x-value: 12
Least y-value: 0
Greatest y-value: 12
Press <ESC>.

Enter the remaining three inequalities and draw the graphs.

Select INEQUALITIES. Use the right-arrow key to fill in the blank parentheses as shown below:

#1: y <= (1)x + (5)

#2: y <= (-0.5)x + (9.5)

#3: y <= (-2)x + (20)

Press <ESC>. Select DRAW.

By inspection, the vertices of the most densely shaded area are: (0, 0), (0, 5), (3, 8), (7, 6), and (10, 0). These are the critical points to test in a linear programming problem. Note that the feasible region has five sides, each of which corresponds to one of the five inequality restrictions.
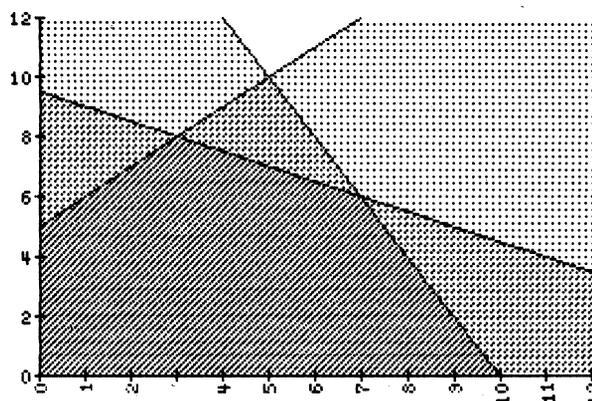


```
y <= (1   )x+(5   )
y <= (-0.5 )x+(9.5 )
y <= (-2   )x+(20  )
```

## Conics Plotter

*Use the Conics Plotter program to investigate systems of ellipses and circles.*

Graph the following system:

$$x^2 + y^2 = 16$$
$$4x^2 + 25y^2 = 100$$

Select Conics Plotter from the main menu. Select EQUATIONS. Fill in the first format for each as follows:

#1: (1)(x − (0))^2 + (1)(y − (0))^2 = (16)

#2: (4)(x − (0))^2 + (25)(y − (0))^2 = (100)

Alternatively, you could fill in the second format for each as follows:

#1: (1)x^2 + (1)y^2 + (0)x + (0)y = (16)

#2: (4)x^2 + (25)y^2 + (0)x + (0)y = (100)

Press ‹ESC›. Select DRAW.



```
(solid)  (1 )(x-(0 ))^2 + (1 )(y-(0 ))^2 = (16 )
(dotted) (4 )(x-(0 ))^2 + (25 )(y-(0 ))^2 = (100 )
```

The first equation represents a circle with radius 4. The second equation represents an ellipse with major axis 10 units and minor axis 4 units. By inspecting the graphs, you can see there are four solutions to the system.

You may find that screen distortion makes circles look like ellipses. You can reduce this distortion by changing the SCALE to display the intervals $-6 \leq x \leq 6$ and $-4 \leq y \leq 4$.

Printouts may be distorted differently than the monitor screen. To achieve an adequate image, you may need to change the scales again. In the example shown, the SCALE was set to display the intervals $-8 \leq x \leq 8$ and $-5 \leq y \leq 5$.

The Conics Plotter program can be used to plot parabolas and lines as well as ellipses and circles. Using the second format, a parabola will result if the coefficient of $x^2$ or $y^2$ is zero. A line will result if both of those coefficients are zero.

# Using the Quizzes

### Line Quiz

for use after Book 1: Lesson 8-5 or Book 2: Lesson 3-4

Select Line Quiz from the main menu.

Select OTHER to choose from the following options.

Difficulty level:  Easy  Hard

Find lines:     As shown  Parallel  Perpendicular

Once the options have been set, press ‹ESC› and select PROBLEM.

If the *As shown* option has been selected, PROBLEM displays a line. You must guess the equation of the displayed line.

Select TRY and fill in the blanks of one of the formats shown below.

y = (   )x + (   )      or  (   )x + (   )y = (   )

Use the right- or left-arrow key to move the cursor. Press ‹RETURN› or ‹ENTER› when you are done.

Whichever linear equation form contains the cursor when ‹RETURN› or ‹ENTER› is pressed will be considered your guess. The computer then graphs the original line and your guess so that you can visually compare the two lines. If the lines do not coincide, you may select TRY again. You may obtain the correct equation by selecting ANSWER at any time.

If the *Parallel* option has been selected, PROBLEM displays a line and a point not on the line. Select TRY and enter a guess for an equation of the line that is parallel to the displayed line and passes through the given point. The computer then graphs the original line and your guess. If the lines do not coincide, you may select TRY again. You may obtain the correct equation by selecting ANSWER at any time.

If the *Perpendicular* option has been selected, PROBLEM displays a line and a point not on the line. Select TRY and enter a guess for an equation of the line that is perpendicular to the displayed line and passes through the given point. The computer then graphs the original line and your guess. If the lines do not coincide, you may select TRY again. You may obtain the correct equation by selecting ANSWER at any time.

### Parabola Quiz

for use after Book 2: Lesson 7-5 or 9-3

Select Parabola Quiz from the main menu.

There are no options to set.

Select PROBLEM.

The computer displays a parabola. You must guess the equation of the displayed parabola.

Select TRY and use the arrow keys to fill in the blanks of the equation format shown below.

y − (   ) = (   )(x − (   ))^2

Press ‹RETURN› or ‹ENTER› when you are done.

The computer will graph the original parabola and your guess so that you can visually compare the two parabolas. If the parabolas do not coincide, you may select TRY again. You may obtain the correct equation by selecting ANSWER at any time.

## Conics Quiz

Select Conics Quiz from the main menu.

Select OTHER to choose from the following options.

Center/Vertex: At origin   Translated
Problem type:   Circles   Parabolas   Ellipses   Hyperbolas   Mixed

Once the option has been set, press ‹ESC› to return to the Conics Quiz menu and select PROBLEM.

The computer graphs a conic of the type selected. You must guess the equation of the conic.

Select TRY and use the arrow keys to fill in the blanks of the equation format shown below.

$$(\quad)(x - (\quad))^\wedge 2 + (\quad)(y - (\quad))^\wedge 2 = (\quad)$$

Press ‹RETURN› or ‹ENTER› when you are done.

The computer will graph the original conic and your guess so that you can visually compare the two conics. If the conics do not coincide, you may select TRY again. You may obtain the correct equation by selecting ANSWER at any time.

## Circular Function Quiz

Select Circular Function Quiz from the main menu.

Select OTHER to choose the difficulty level and type of circular function from the following options.

Difficulty level:   Easy   Moderate   Hard
Problem type:   Sin, Cos   Tan, Cot   Sec, Csc

Once the options have been set, press ‹ESC› and select PROBLEM.

The computer graphs a circular function of the type selected. You must guess the equation of the function.

Select TRY and type in the name of the function. Use parentheses around the argument of a circular function. Type PI for $\pi$. An example is shown below.

y = 2SIN(X – PI/2) + 1

Press ‹RETURN› or ‹ENTER› when you are done.

The computer will graph the original circular function and your guess so that you can visually compare the two graphs. If the graphs do not coincide, you may select TRY again. You may obtain the correct equation by selecting ANSWER at any time.

---

# Using the Utilities

## Matrix Reducer

*Use the Matrix Reducer to solve a system of linear equations in three variables.*

Consider the system of equations shown at the right.

$$x + 2y + 3z = 3$$
$$2x + 3y + 4z = 2$$
$$-3x - 5y + 2z = 4$$

Create a matrix using the coefficients in the given system.

Select Matrix Reducer from the main menu.

Select NEW MATRIX and highlight two numbers to create a 3 by 4 matrix.

Number of rows: 2 $\underline{3}$ 4 5

Number of columns: 2 3 $\underline{4}$ 5 6

Select CHANGE MATRIX and enter the coefficients, using arrow keys to move through the matrix. Press ‹ESC› when done.

$$\begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 3 & 4 & 2 \\ -3 & -5 & 2 & 4 \end{bmatrix}$$

Select REDUCE MATRIX.

The following options appear:

Add ( ) row ( ) to ( ) row ( )
Multiply row ( ) by ( )
Interchange row ( ) and row ( )
Back up one step

The object is to use row operations to transform the matrix into triangular form (that is, with zeros in the lower left corner). The matrices shown at the right below are the results of the row operations listed at the left.

Add (2) row (1) to (−1) row (2)

This replaces the second row with the sum of 2 times the first row and −1 times the second row.

$$\begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 1 & 2 & 4 \\ -3 & -5 & 2 & 4 \end{bmatrix}$$

Add (3) row (1) to (1) row (3)

This replaces the third row with the sum of 3 times the first row and the third row.

$$\begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & 11 & 13 \end{bmatrix}$$

Add (−1) row (2) to (1) row (3)

This replaces the third row with the sum of −1 times the second row and the third row.

$$\begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 9 & 9 \end{bmatrix}$$

The resulting matrix is in triangular form. The third row tells you that $9z = 9$, so $z = 1$. The second row tells you that $1y + 2z = 4$. Since $z = 1$, you have $1y + 2(1) = 4$, or $y = 2$. The first row tells you that $1x + 2y + 3z = 3$. Since $y = 2$ and $z = 1$, you have $1x + 2(2) + 3(1) = 3$, or $x = -4$.

Therefore, the solution of the system is $(-4, 2, 1)$.

## Statistics Spreadsheet

*Use the Statistics Spreadsheet program to explore heartbeat data and plot the data as a histogram and scatterplot.*

Select Statistics Spreadsheet from the main menu.

Create a spreadsheet for data entry.

Select ENTER DATA.

Enter names in each row of the numbered list, and heartrates before exercise (Column A) and after exercise (Column B).

Use tab and arrow keys to move in the spreadsheet. Enter the data shown at the right.

Press ‹ESC› when all data are entered.

The program will calculate the least and greatest values in each column, as well as the median, mean, and standard deviation of each column.

Select CALCULATE.

The mean for the *Before* column is 84.1, with values ranging from 69 to 97; the mean for the *After* column is 153.9, with values ranging from 144 to 175.

You can present the data of either column as a histogram.

Select GRAPH. Select HISTOGRAM.

The first column is automatically highlighted. Press ↵ to accept this column. Its least value is 69. Instruct the computer to group the data in intervals of five.

Least value in first group: 65
Least value in second group: 70
Press ↵.

The histogram shows the frequency of values within each interval of five.

A scatterplot of the two columns can be created.

Press ‹ESC›. Select SCATTER.

Each x represents a data pair corresponding to a student's test scores. A line of best fit is drawn. Press the space bar to see the correlation coefficient for the data pairs, and the equation of the line of best fit:

$y = (.80755805)x + (85.984368)$.

| Item Label | Before | After |
|---|---|---|
| 1. Joe | 69 | 145 |
| 2. Alexa | 75 | 153 |
| 3. Jean | 88 | 161 |
| 4. Lamont | 97 | 164 |
| 5. Tanya | 80 | 144 |
| 6. Tim | 88 | 152 |
| 7. Jonah | 84 | 144 |
| 8. Ruth | 78 | 148 |
| 9. Kevin | 92 | 175 |
| 10. Maria | 90 | 153 |
| Number | 10 | 10 |
| Least | 69 | 144 |
| Greatest | 97 | 175 |
| Median | 86 | 152.5 |
| Mean | 84.1 | 153.9 |
| Std Dev | 8.1172655 | 9.5545802 |
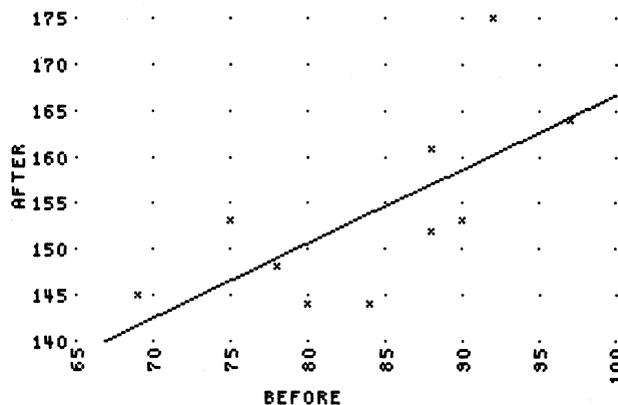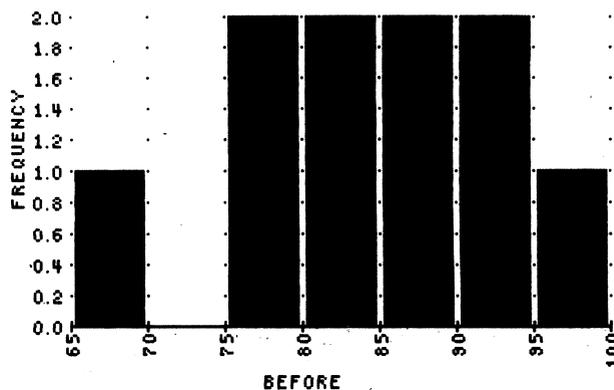
## Sampling Experiment

*Investigate the statistical characteristics of samples.*

Select Sampling Experiment from the main menu. Select DEFINE.

The computer will create in its memory a "population" of normally distributed numbers having certain default characteristics: a mean of 100 and a standard deviation of 10. Note that the sample size is 20; when the computer samples the population, it will randomly select 20 "individuals" from it.

What do you think the mean of the sample will be? You should not expect the mean to be exactly 100, but you should expect the mean to be near 100. Use SINGLE SAMPLE to confirm or deny this.

Select SINGLE SAMPLE.

Repeat by selecting SINGLE SAMPLE several times. A histogram showing the frequency of the sample means obtained from different samples should approximate a normal curve, clustered about the population mean. The MANY SAMPLES program creates such a histogram.

Select MANY SAMPLES. Allow it to run for at least 100 samples.

Experiment with various sample sizes to see how the sample size affects the shape of the histogram of sample means. You may wish to compare printouts of histograms of 200 sample means each for sample sizes of 20, 10, and 5. [See diagrams on next page.]

Select DEFINE. Change the sample size. Select MANY SAMPLES.

What do you notice about the histograms? [The smaller the sample size is, the more dispersed the sample means are.] What implication does this have for statisticians in the real world? [A sample that is too small may produce a statistic that does not reasonably characterize the population.]

Select DEFINE and change the numbers to:

|  |  |  |  |
|---|---|---|---|
| Population mean: | 180 | Value to test: | 195 |
| Population standard dev.: | 8 | Least sample mean shown: | 150 |
| Sample size: | 1 | Greatest sample mean shown: | 210 |

What is true about the mean of a sample when the sample size is 1? [The number is the mean of the sample.] Select MANY SAMPLES, and let the computer create a histogram of about 700 sample means (which also represent 700 individuals from the population, since the sample size is 1). [See diagram on next page.] What percent of the sample means (and therefore individuals) is less than 195? [About 96.31%] What percent would therefore be greater than 195? [About 3.69%]

**Sample size: 20**
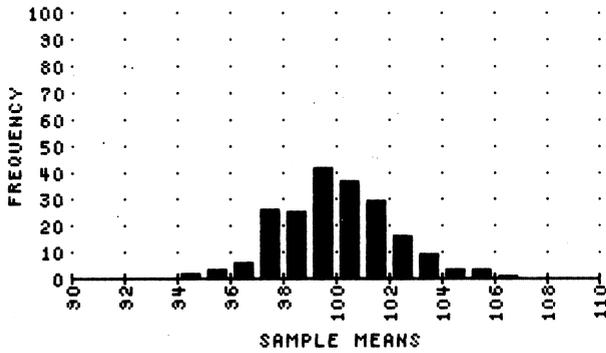


**Sample size: 1**
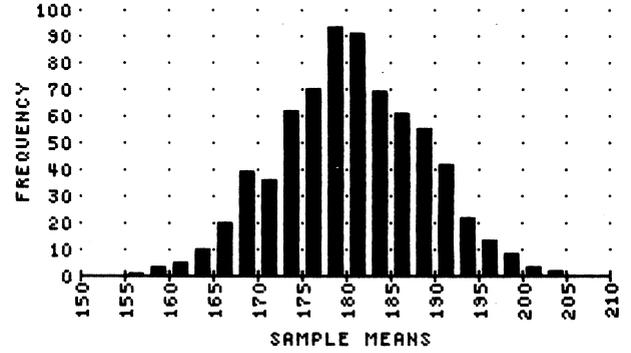


**Sample size: 10**
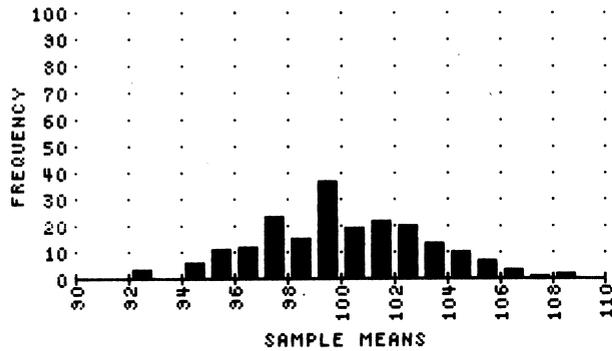


| Sample Mean | % of Samples with Smaller Mean | % of Samples with Larger Mean |
|---|---|---|
| 150.0 | 0% | 100% |
| 152.5 | 0% | 100% |
| 155.0 | 0% | 100% |
| 157.5 | .14184397% | 99.858156% |
| 160.0 | .56737589% | 99.432624% |
| 162.5 | 1.2765957% | 98.723404% |
| 165.0 | 2.6950355% | 97.304965% |
| 167.5 | 5.5319149% | 94.468085% |
| 170.0 | 11.06383% | 88.93617% |
| 172.5 | 16.170213% | 83.829787% |
| 175.0 | 24.964539% | 75.035461% |
| 177.5 | 34.893617% | 65.106383% |
| 180.0 | 48.085106% | 51.914894% |
| 182.5 | 60.992908% | 39.007092% |
| 185.0 | 70.780142% | 29.219858% |
| 187.5 | 79.432624% | 20.567376% |
| 190.0 | 87.234042% | 12.765957% |
| 192.5 | 93.191489% | 6.8085106% |
| 195.0 | 96.312057% | 3.6879432% |
| 197.5 | 98.156028% | 1.8439716% |
| 200.0 | 99.29078% | .70921987% |
| 202.5 | 99.716312% | .28368795% |
| 205.0 | 100% | 0% |
| 207.5 | 100% | 0% |
| 210.0 | 100% | 0% |

Percent less than 195: 96.312057%
Sample size: 1

**Sample size: 5**
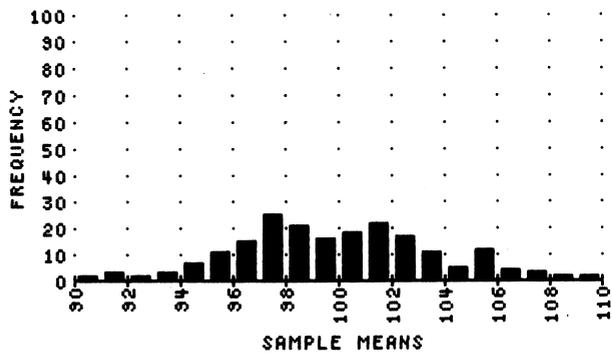
# Correlation to Algebra, Structure and Method, Book 1

| Lesson | Scripted Demo | Enrichment Topic | Activity Sheet | Title |
|---|---|---|---|---|
| 8-2 | | | p. 43 | Graphs of Linear Equations in Standard Form |
| 8-3 | pp. 18–21 | | | Slope of a Line |
| 8-4 | | | p. 44 | Slope-Intercept Form |
| 8-8 | | | p. 45 | Zeros of Functions |
| 8-9 | | p. 31 | | Determining Direct Variation |
| 8-10 | | | p. 46 | Inverse Variation |
| 9-1 | pp. 21–22 | | | The Graphing Method |
| 9-1 | | | p. 47 | Solving Systems of Linear Equations |
| 9-4 | | pp. 32–33 | | The Addition-or-Subtraction Method |
| 10-7 | pp. 22–24 | | | Graphing Linear Inequalities |
| 10-7 | | | p. 48 | Absolute Value Graphs |
| 10-8 | | pp. 33–34 | | Real-World Applications of Linear Inequalities |
| 10-8 | | | p. 49 | Linear Programming |
| 11-4 | | p. 34 | | Finding Irrational Square Roots by Graphing |
| 11-10 | | p. 35 | | Solving Simple Radical Equations by Graphing |
| 12-1 | | p. 35 | | Quadratic Equations with Perfect Squares |
| 12-4 | pp. 24–25 | | | Quadratic Equations: The Discriminant |

Page references in the charts on these two pages refer to this booklet.

The lesson numbers in the first column of each chart refer to *Algebra, Structure and Method, Book 1* and *Algebra and Trigonometry, Structure and Method, Book 2* by Brown, Dolciani, Sorgenfrey, Cole, and Kane (Houghton Mifflin Company, © 1990).

# Correlation to Algebra and Trigonometry, Structure and Method, Book 2

| Lesson | Scripted Demo | Enrichment Topic | Activity Sheet | Title |
|--------|---------------|------------------|----------------|-------|
| 3-2 | | pp. 35–36 | | Diophantine Equations |
| 3-3 | | | p. 50 | Exploring Slopes |
| 3-4 | | | p. 51 | Parallel and Perpendicular Lines |
| 3-5 | | | p. 52 | Estimating Solutions of Linear Systems |
| 3-7 | pp. 26–27 | | | Linear Inequalities in Two Variables |
| 3-8 | | p. 36 | | Determining the Domain and Range of Functions by Graphing |
| 3-10 | | p. 37 | | Solving Absolute Value Inequalities |
| 4-7 | | p. 37 | | Solving Polynomial Equations by Graphing |
| 6-1 | | p. 37 | | Graphing to Find $n$th Roots of Real Numbers |
| 7-5 | | | p. 53 | Parabolas |
| 7-6 | pp. 27–30 | | | Quadratic Functions |
| 8-1 | | p. 38 | | Curve Fitting |
| 8-8 | | pp. 38–39 | | Approximations with Greater Accuracy |
| 8-8 | | | p. 54 | Approximating Roots of Polynomial Equations |
| 9-3 | | p. 39 | | Exploring Parabolas |
| 9-6 | | | p. 55 | Identifying Central Conics |
| 9-7 | | | p. 56 | Estimating Solutions of Quadratic Systems |
| 10-2 | | | pp. 57–58 | Exponential Growth |
| 10-3 | | p. 39 | | Functions with Inverses on a Restricted Domain |
| 10-3 | | | p. 59 | Functions and Their Inverses |
| 10-4 | | pp. 39–40 | | Graphing Logarithmic Equations |
| 10-4 | | p. 40 | | Curve Fitting Using Exponential Functions |
| 13-4 | | pp. 40–41 | | Graphing the Taylor Series for Sine |
| 13-4 | | | pp. 60–61 | Sine and Cosine Curves |
| 13-5 | | | pp. 62–63 | Tangent, Cotangent, Secant, and Cosecant |
| 13-9 | | p. 41 | | Verifying Trigonometric Identities by Graphing |
| 14-8 | | p. 41 | | Solving Trigonometric Equations by Graphing |
| 15-2 | | | p. 64 | Histograms, Means, and Standard Deviation |
| 15-4 | | | p. 65 | Correlation and Line of Best Fit |
| 15-9 | | | p. 66 | Sampling |
| 16-6 | | p. 42 | | Finding the Inverse of a Matrix |

# Scripted Demonstrations

In the following scripted demonstrations, T refers to statements and questions a teacher might offer, while S refers to typical student responses that might be expected.

## Book 1, Lesson 8-3 • Slope of a Line

**T1.** Use Line Plotter to graph the lines $y = 1x$ and $y = 2x$. (The line $y = 1x$ will appear solid; the line $y = 2x$ will appear dotted.)

**S1.** [See diagram.]

**T2.** Notice that both lines pass through the origin and both rise from left to right. What makes the lines different?

**S2.** One line is steeper than the other.

**T3.** What is the equation of the steeper line?

**S3.** $y = 2x$

**T4.** Place your finger at the origin. If you move 1 unit to the *right* along the x-axis), how far *up* must you move to get to the line:

   **a.** $y = 1x$?        **b.** $y = 2x$?

**S4.** **a.** 1 unit       **b.** 2 units

**T5.** Slope is a measure of the steepness of a line. It is defined as the ratio of vertical change to horizontal change as you move from one point to another point on a line. For example, the slope of $y = 1x$ is $\frac{1}{1}$, or 1, because you must move 1 unit up to get back to the line after moving 1 unit to the right from the origin. What, then, is the slope of the line $y = 2x$?

**S5.** $\frac{2}{1}$, or 2

**T6.** The slope of the line $y = 1x$ is 1. The slope of the line $y = 2x$ is 2. What would you expect the slope of the line $y = 3x$ to be?

**S6.** 3

**T7.** In the first quadrant, would you expect the graph of $y = 3x$ to lie above, between, or below the graphs of $y = 1x$ and $y = 2x$?

**S7.** Above

**T8.** Confirm your expectation by graphing $y = 3x$. (The line will appear dashed.)

**S8.** [See diagram.]

**T9.** If you were to graph lines with greater slopes, how would they appear?

**S9.** The lines would become more vertical.
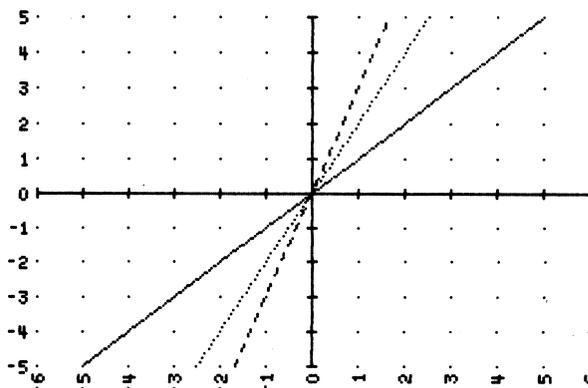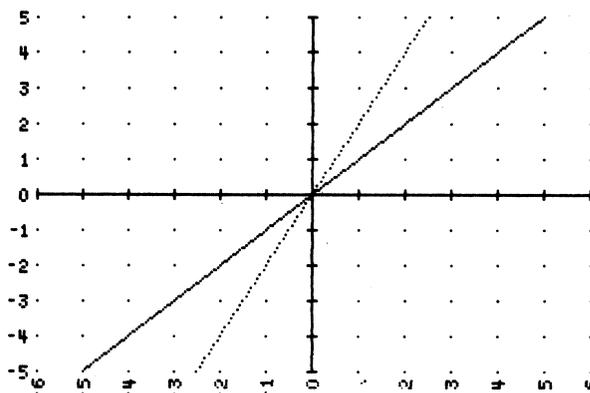
**T10.** Look at the *y*-axis, which is a vertical line. As you move from the origin to another point on the line, say (0, 1), what is the amount of:

    **a.** horizontal change?

    **b.** vertical change?

**S10.** **a.** 0

    **b.** 1

**T11.** Since slope is the ratio of vertical change to horizontal change, what can we say about the slope of a vertical line like the *y*-axis?

**S11.** The ratio $\dfrac{1}{0}$ is undefined, so the slope of a vertical line is undefined.

**T12.** In fact, we say that a vertical line has *no* slope. Now let's consider a line with slope between 0 and 1. The line $y = \dfrac{1}{2}x$ has what slope?

**S12.** $\dfrac{1}{2}$

**T13.** How will the graph of $y = \dfrac{1}{2}x$ compare with the lines already graphed?

**S13.** Because the graph of $y = \dfrac{1}{2}x$ has a smaller slope than the other lines, it is less steep and therefore lies below the other lines in the first quadrant.

**T14.** Confirm your prediction by graphing $y = \dfrac{1}{2}x$. (The line will appear dashed.)

**S14.** [See diagram.]

**T15.** If you were to graph lines with smaller (but still positive) slopes, how would they appear?

**S15.** The lines would become more horizontal.

**T16.** Look at the *x*-axis, which is a horizontal line. As you move from the origin to another point on the line, say (1, 0), what is the amount of:

    **a.** horizontal change?

    **b.** vertical change?

**S16.** **a.** 1

    **b.** 0

**T17.** Since slope is the ratio of vertical change to horizontal change, what can you say about the slope of a horizontal line like the *x*-axis?

**S17.** The ratio $\dfrac{0}{1}$ equals 0, so the slope of a horizontal line is 0.

**T18.** Let's summarize what we've discussed so far. Since slope is a measure of the steepness of a line, how would you describe a line that has:

    **a.** no slope?     **b.** 0 slope?     **c.** positive slope?

**S18.** **a.** It is vertical.     **b.** It is horizontal.     **c.** It is a slanted line.
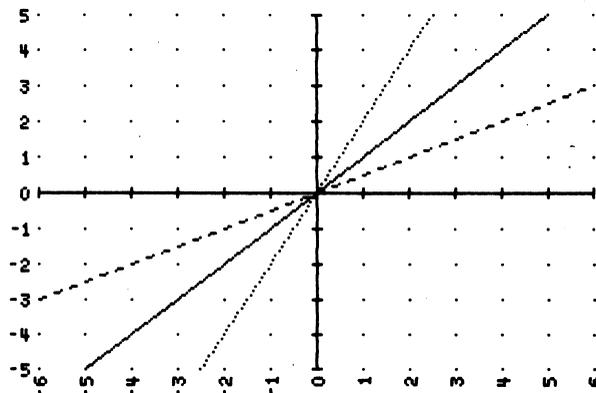
**T19.** Although lines with positive slope are indeed slanted, the converse is not true. That is, not all slanted lines have positive slope. Some have negative slope. To investigate negative slope, graph the lines $y = 1x$ and $y = -1x$. (The line $y = 1x$ will appear solid; the line $y = -1x$ will appear dotted.)

**S19.** [See diagram.]

**T20.** Both lines pass through the origin. Place your finger there. If you move 1 unit to the right (along the $x$-axis), how must you move vertically to get back to the line:

    **a.** $y = 1x$?        **b.** $y = -1x$?

**S20.** **a.** 1 unit up      **b.** 1 unit down

**T21.** To indicate a downward movement, we use a negative sign. So there is a vertical change of $-1$ unit for each horizontal change of 1 unit on the line $y = -1x$. This means that the slope of the line is $\frac{-1}{1}$, or $-1$. What do you expect the slope of the line $y = -2x$ to be?

**S21.** $-2$

**T22.** In the fourth quadrant, would you expect the graph of $y = -2x$ to lie above or below the graph of $y = -1x$?

**S22.** Below

**T23.** Confirm your expectation by graphing $y = -2x$. (The line will appear dashed.)

**S23.** [See diagram.]

**T24.** What would be an equation of a line that passes through the origin, has negative slope, and lies above the line $y = -1x$ in the fourth quadrant?

**S24.** [Answers may vary. For example: $y = -\frac{1}{2}x$]

**T25.** Confirm your answer by graphing the equation.

**S25.** [See diagram.]
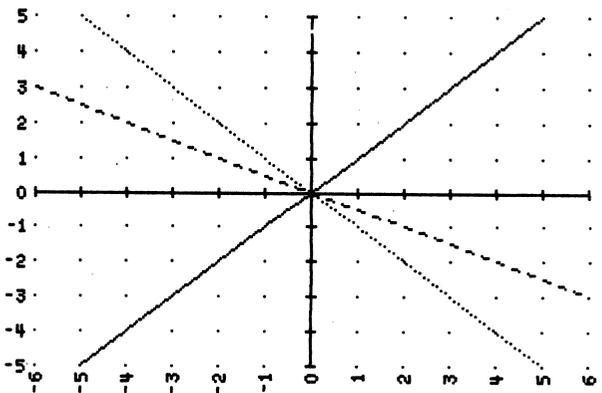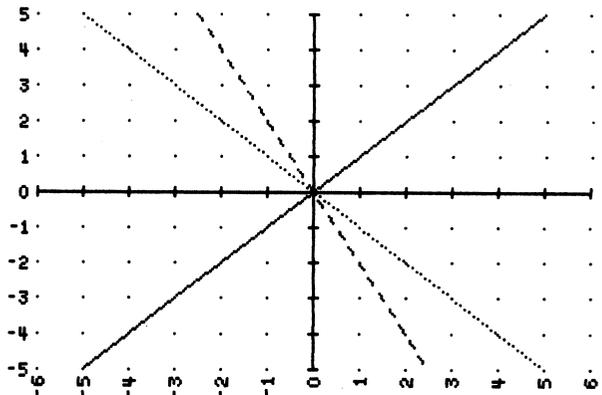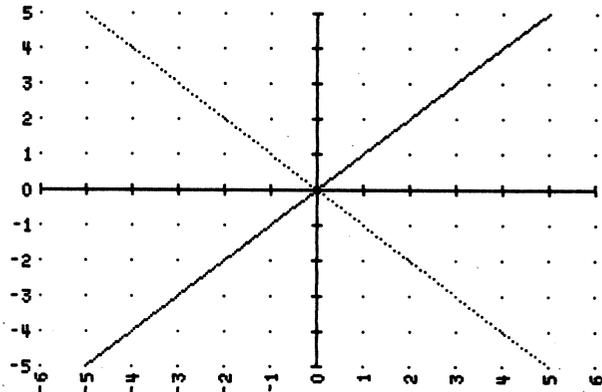
**T26.** Let's summarize all that we've discussed.

    **a.** Since slope is a measure of the steepness of a line, a line that is "flat" (that is, horizontal) has what slope?

    **b.** With regard to slope, how are slanted lines that *rise* from left to right distinguished from slanted lines that *fall*?

    **c.** Suppose two lines both rise or both fall from left to right. With regard to slope, how are the lines distinguished?

    **d.** What do we say about the slope of a vertical line?

**S26. a.** 0

    **b.** Rising lines have positive slopes; falling lines have negative slopes.

    **c.** The absolute value of the slope of the steeper line (that is, the line that is more nearly vertical) is greater.

    **d.** A vertical line has no slope.

### Book 1, Lesson 9-1 • The Graphing Method

**T1.** Use Line Plotter to graph the lines $2x - 3y = 6$ and $4x - 6y = 3$.

**S1.** [See diagram.]

**T2.** If you were asked to determine the coordinates of any points that the two lines have in common, what would you say?

**S2.** The two lines are parallel, so they don't have any points in common.
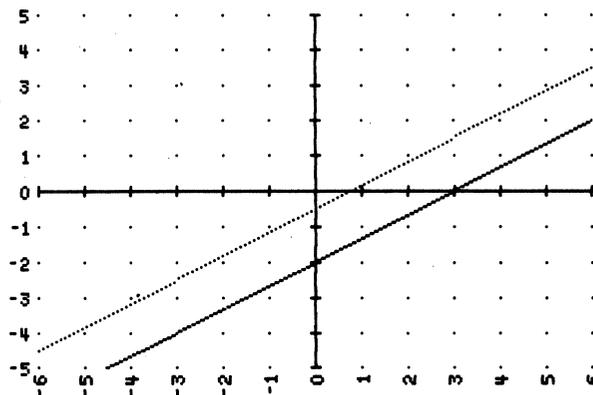
**T3.** Now graph the lines $2x - 3y = 6$ and $-4x + 6y = -12$.

**S3.** [See diagram.]

**T4.** What do you notice about the intersection of these two lines?

**S4.** The two lines coincide, so there are infinitely many points of intersection.
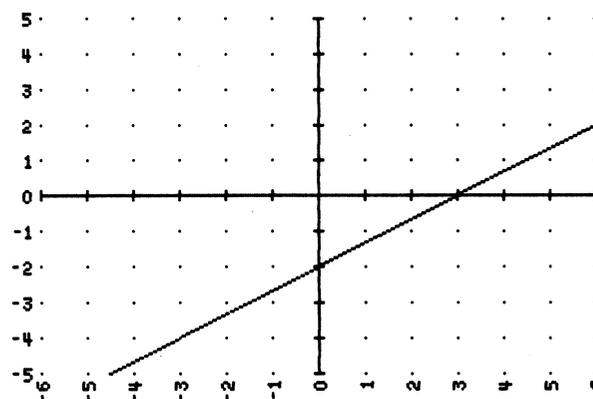
**T5.** Now graph the lines $2x - 3y = 6$ and $4x + 6y = 3$.

**S5.** [See diagram.]

**T6.** These two lines intersect in a single point. The graph's current scale allows you to estimate the coordinates of this point only to the nearest integer. What is your estimate of the coordinates?
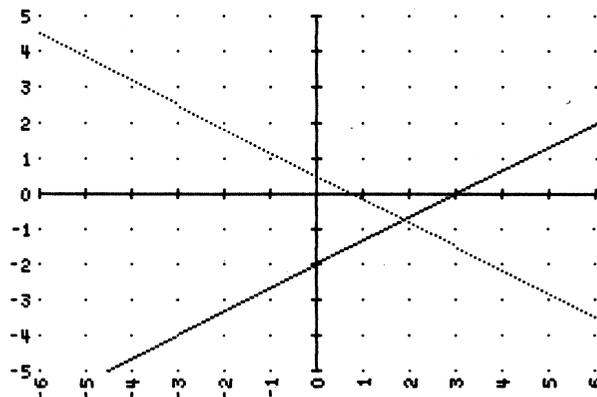
**S6.** $(2, -1)$

**T7.** You can get a better estimate of the coordinates by zooming in on the point of intersection. Do so.
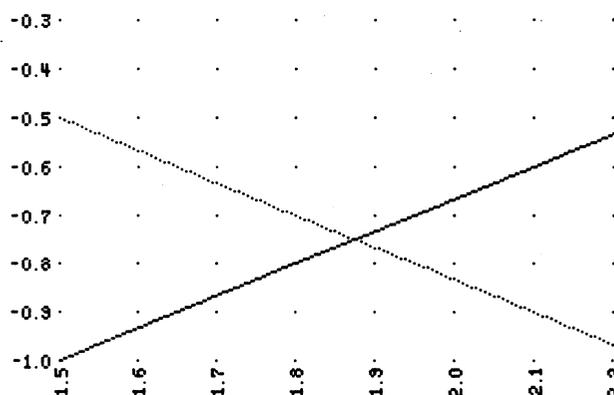
**S7.** [See diagram.]

**T8.** To the nearest tenth, what are the coordinates of the point of intersection?

**S8.** $(1.9, -0.8)$

**T9.** Let's summarize what we've discussed. When you're given the equations of two lines and asked to find any points of intersection, what can happen?

**S9.** The two lines can be parallel, in which case there is no point of intersection. Or the two lines can coincide, in which case there are infinitely many points of intersection. Or the two lines can intersect in a single point, the coordinates of which can be estimated from the graph (with better approximations obtained by zooming).

## Book 1, Lesson 10-7 • Graphing Linear Inequalities

**T1.** You already know that the graph of the equation $y = x$ is a line. What does the graph of the inequality $y > x$ look like? To find out, use Inequality Plotter.

**S1.** [See diagram.]

**T2.** Notice that the line $y = x$, which is dotted, divides the coordinate plane into two regions called *half-planes*. Give the coordinates of a point in the shaded half-plane.
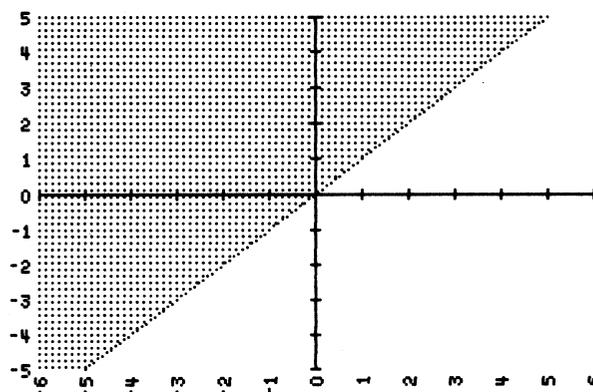
**S2.** [Answers will vary. For example: (0, 1)]

**T3.** Is the $y$-coordinate of this point greater than or less than the $x$-coordinate?

**S3.** Greater

**T4.** Give the coordinates of a point in the unshaded half-plane.

**S4.** [Answers will vary. For example: (1, 0)]

**T5.** Is the $y$-coordinate of this point greater than or less than the $x$-coordinate?

**S5.** Less

**T6.** Why is the half-plane above the line $y = x$ shaded?

**S6.** Every point above the line $y = x$ has a $y$-coordinate that is greater than the $x$-coordinate.

**T7.** What would the graph of $y < x$ look like?

**S7.** The half-plane below the line $y = x$ would be shaded.

**T8.** Confirm your prediction by graphing $y < x$.

**S8.** [See diagram.]

**T9.** How do you think the graph of $y \leq x$ would differ from the graph of $y < x$? Why?

**S9.** The graph would be the same, except that the line $y = x$ would be solid. This time the points on the line are part of the solution.

**T10.** Confirm your prediction by graphing $y \leq x$.

**S10.** [See diagram.]
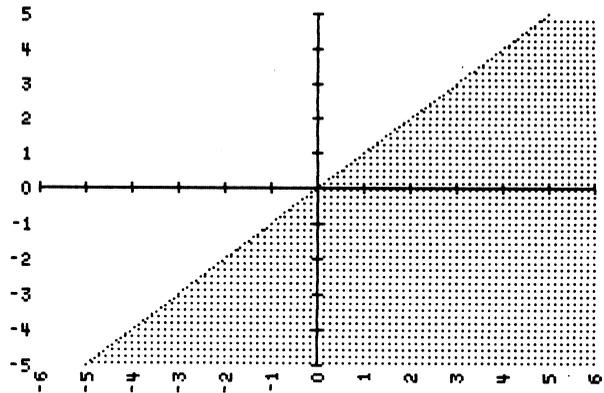
**T11.** Get a piece of graph paper and draw and label the coordinate axes. If we wanted to graph the inequality $x - y > 3$ ourselves, what is the first thing we should do?

**S11.** Graph the line $x - y = 3$.

**T12.** Would we make the line solid or not? (By the way, although the computer draws a non-solid line dotted, we will draw such a line dashed instead.)

**S12.** The line should be dashed, because equality isn't involved.

**T13.** Go ahead and draw the dashed line on your graph paper. What's the next step?

**S13.** Shade.

**T14.** How do we know which half-plane to shade?

**S14.** Pick a point and see if its coordinates satisfy the inequality.

**T15.** The point we choose should not be on the line. Let's use a convenient point such as the origin. What are the coordinates of the origin? Do they satisfy the inequality $x - y > 3$?

**S15.** The origin has coordinates (0, 0). They don't satisfy the inequality, because $0 - 0$ isn't greater than 3.

**T16.** So what does that mean in terms of shading the graph?

**S16.** We should shade the half-plane that *doesn't* contain the origin.
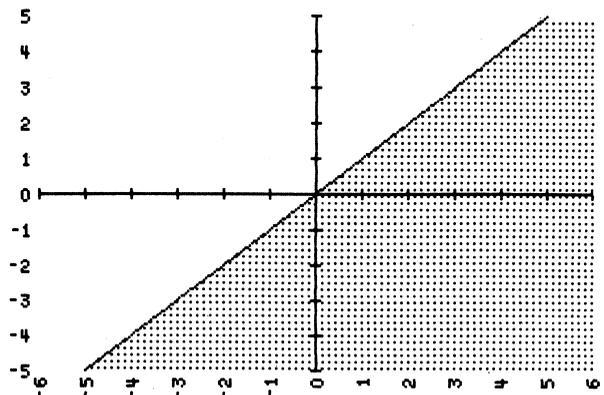
**T17.** Go ahead and shade your graph. Verify your graph by graphing $x - y > 3$ on the computer.

**S17.** [See diagram.]

**T18.** Let's summarize what we've discussed. The graph of a linear inequality is a half-plane. What are the steps needed to obtain the graph?

**S18.** First graph the line, making it solid or dashed depending on whether the inequality does or does not include equality. Then choose a convenient point not on the line and check to see whether its coordinates satisfy the inequality. If they do, shade the half-plane containing the point; if they don't, shade the half-plane on the other side of the line.
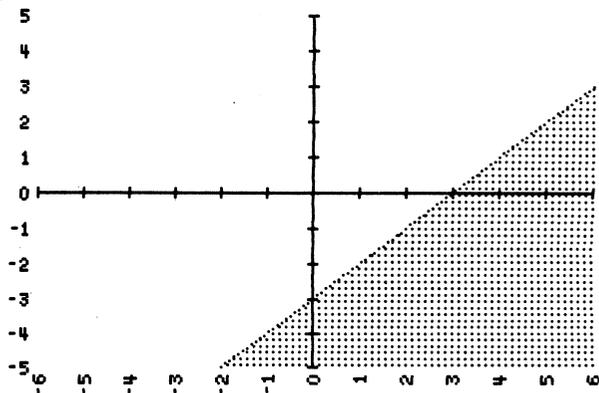
## Book 1, Lesson 12-4 • Quadratic Equations: The Discriminant

**T1.** As you already know, the quadratic formula gives the roots of the quadratic equation $ax^2 + bx + c = 0$ in terms of $a$, $b$, and $c$. State the quadratic formula.

**S1.** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**T2.** The portion of the formula that appears under the radical sign, $b^2 - 4ac$, plays an important role in determining whether the equation $ax^2 + bx + c = 0$ has two, one, or no real roots. We call $b^2 - 4ac$ the *discriminant* of the equation. One way to see how the discriminant indicates the number of real roots is to look at a quadratic equation geometrically. Recall that the graph of $y = ax^2 + bx + c$ is a parabola. Let's use Parabola Plotter to graph the equation $y = x^2 - 4x + 3$.

**S2.** [See diagram.]

**T3.** The $x$-coordinate of a point where a curve crosses the $x$-axis is called an *x-intercept* of the curve. What are the $x$-intercepts of the parabola $y = x^2 - 4x + 3$?

**S3.** 1 and 3

**T4.** Notice that these $x$-intercepts are the solutions of the equation $x^2 - 4x + 3 = 0$. Why should this be the case?

**S4.** The $y$-coordinate of any point on the $x$-axis is 0. Therefore the equation $y = x^2 - 4x + 3$ becomes $0 = x^2 - 4x + 3$ on the $x$-axis.
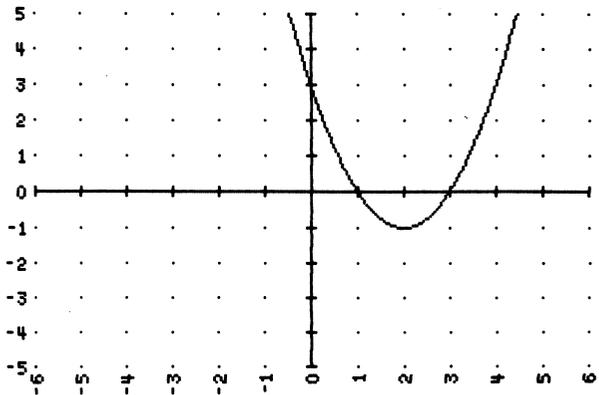
**T5.** For the record, what value does the discriminant of $x^2 - 4x + 3 = 0$ have?

**S5.** $(-4)^2 - 4(1)(3) = 4$

**T6.** Now graph the parabola $y = x^2 - 4x + 4$. (It will appear dotted.)

**S6.** [See diagram.]

**T7. a.** What are the solutions of the equation $x^2 - 4x + 4 = 0$? (What are the $x$-intercepts of the parabola $y = x^2 - 4x + 4$?)

    **b.** What value does the discriminant have?

**S7. a.** The only solution is 2.

    **b.** $(-4)^2 - 4(1)(4) = 0$

**T8.** Now graph the parabola $y = x^2 - 4x + 5$. (It will appear dashed.)

**S8.** [See diagram again.]

**T9. a.** Use the graph to determine the solutions of $x^2 - 4x + 5 = 0$. What are they?

    **b.** What value does the discriminant have?

**S9. a.** The graph does not cross the $x$-axis; the equation has no real solutions.

    **b.** $(-4)^2 - 4(1)(5) = -4$

**T10.** To summarize:

    **a.** $x^2 - 4x + 3 = 0$ has two real roots, and the value of the discriminant is 4.

    **b.** $x^2 - 4x + 4 = 0$ has one real root, and the value of the discriminant is 0.

    **c.** $x^2 - 4x + 5 = 0$ has no real roots, and the value of the discriminant is $-4$.

Now graph these three equations:
$y = x^2 + 6x + 7$, $y = x^2 + 6x + 9$, and $y = x^2 + 6x + 11$.

**S10.** [See diagram.]

**T11.** For each of the equations $x^2 + 6x + 7 = 0$, $x^2 + 6x + 9 = 0$, and $x^2 + 6x + 11 = 0$, what are the value of the discriminant and the number of real roots?

**S11.** For $x^2 + 6x + 7 = 0$, the value of the discriminant is 8, and there are two real roots. For $x^2 + 6x + 9 = 0$, the value of the discriminant is 0, and there is one real root. For $x^2 + 6x + 11 = 0$, the value of the discriminant is $-8$, and there are no real roots.

**T12.** What relationship seems to exist between the value of the discriminant and the number of real roots of a quadratic equation?

**S12.** A positive discriminant indicates two real roots, a zero discriminant indicates one real root, and a negative discriminant indicates no real roots.
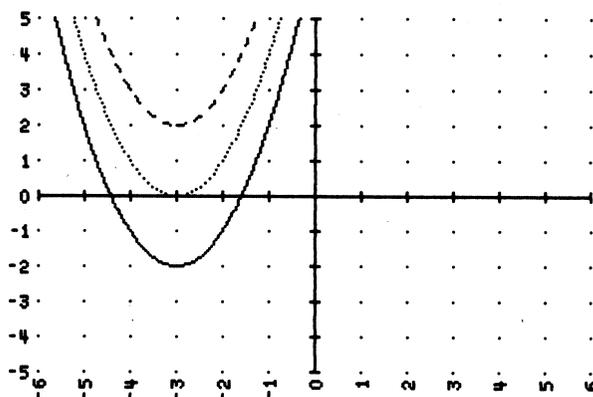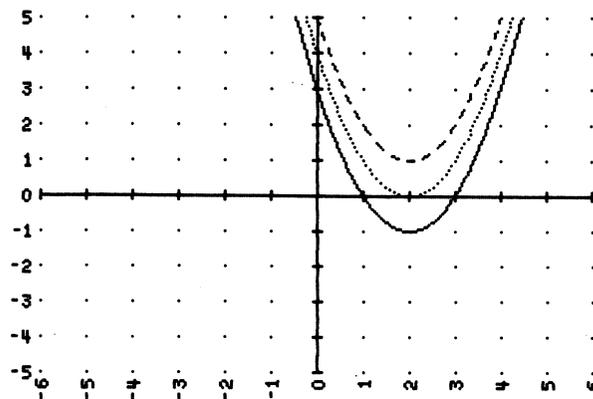
**T1.** Recall that the graph of a linear inequality is a half-plane. What are the steps needed to obtain the graph of a linear inequality?

**S1.** First graph the line, making it solid or dashed depending on whether the inequality does or does not include equality. Then choose a convenient point not on the line and check to see whether its coordinates satisfy the inequality. If they do, shade the half-plane containing the point; if they don't, shade the half-plane on the other side of the line.

**T2.** Now let's consider a *system* of linear inequalities. Graph this system on the computer:

$$2x + 3y \leq 6$$
$$y \geq x + 2$$

**S2.** [See diagram.]

**T3.** We're interested only in the points whose coordinates satisfy *both* inequalities. How can we tell from the graph where those points are?

**S3.** They are in the region that is doubly shaded.

**T4.** Notice that this region is to the left of the point of intersection of the lines $2x + 3y = 6$ and $y = x + 2$. Suppose we change the system by reversing the inequality sign of the first inequality. That is, we now want to graph this system:
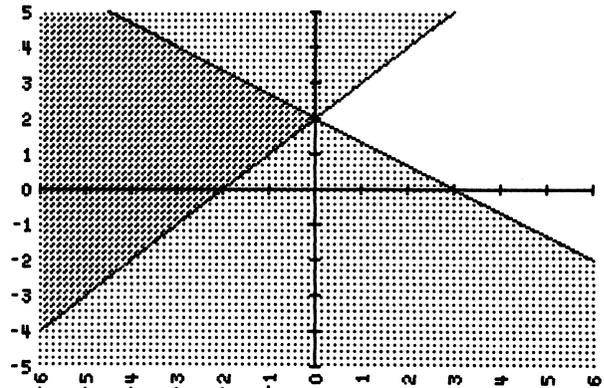
$$2x + 3y \geq 6$$
$$y \geq x + 2$$

How would the graph of the new system differ from the graph of the old one?

**S4.** By reversing the inequality sign of the first inequality, we must switch the shading from below the line $2x + 3y = 6$ to above it. So the double shading for the system would switch from being to the left of the intersection point to being above it.

**T5.** Confirm your prediction by graphing the new system.

**S5.** [See diagram.]

**T6.** What system would have a graph that is the region *below* the intersection of the lines $2x + 3y = 6$ and $y = x + 2$?

**S6.** $2x + 3y \leq 6$
$y \leq x + 2$
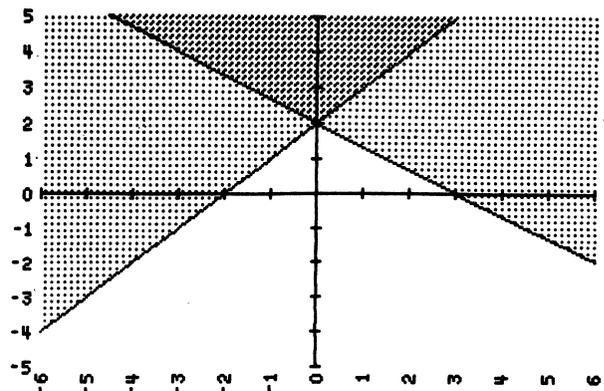
**T7.** Confirm your prediction by graphing the system.

**S7.** [See diagram.]

**T8.** Suppose we add a third inequality, say $y \geq 0$, to the system. How would the graph of the system change?

**S8.** Since the inequality $y \geq 0$ means that the shading is above the $x$-axis, the graph of the system would become the triangular region with vertices at $(-2, 0)$, $(0, 2)$, and $(3, 0)$.

**T9.** Confirm your prediction by graphing:

$$2x + 3y \leq 6$$
$$y \leq x + 2$$
$$y \geq 0$$

**S9.** [See diagram.]

**T10.** Let's summarize what we've discussed about systems of linear inequalities. How do we obtain the graph of a given system?

**S10.** We graph each of the system's inequalities separately and then look for the region where the shadings overlap. This region is the graph of the system.
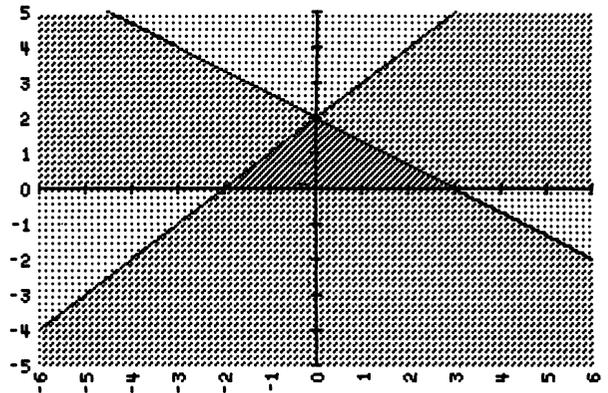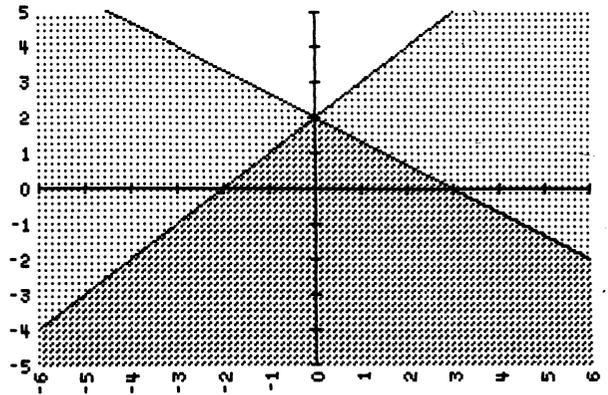
## Book 2, Lesson 7-6 • Quadratic Functions

**T1.** As you already know, the graph of a function like $f(x) = 4x$ is a line. Suppose we alter the function by introducing a second-degree, or quadratic, term like $x^2$. Although the function $f(x) = 4x$ is called linear, the function $g(x) = x^2 + 4x$ is called *quadratic*. What does the graph of $g$ look like? To find out, use Function Plotter to graph $y = x^2 + 4x$.

**S1.** [See diagram.]

**T2.** What do you notice about the shape of the graph?

**S2.** It is a parabola.

**T3.** Since the graph of a quadratic function is a parabola, the vertex of the parabola is of special interest to us because at the vertex the function takes on its *minimum* value (if the parabola opens upward, as the graph of $g$ does) or its *maximum* value (if the parabola opens downward). Look at the graph of $g$ and tell me:

    **a.** For what value of $x$ does $g$ take on its minimum value?

    **b.** What is the minimum value of $g$? (*Hint:* It's the $y$-coordinate of the vertex.)
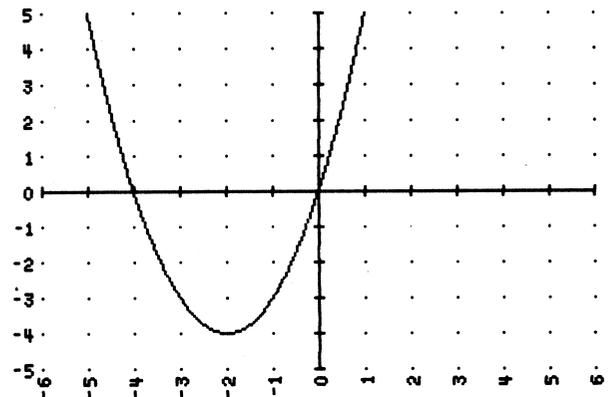
**S3.** **a.** $-2$

    **b.** $-4$

**T4.** We would like to know where a quadratic function takes on its minimum or maximum value (as well as what the minimum or maximum value is) *without* graphing the function. This can be done if we first examine the zeros of the function. Recall that a *zero* of a function $f$ is any value of $x$ for which $f(x) = 0$. The zeros of $g(x) = x^2 + 4x$ are 0 and $-4$. How can you determine the zeros of $g(x) = x^2 + 4x$:

   **a.** by looking at the graph of $g$?

   **b.** without looking at the graph of $g$?

**S4.** **a.** The zeros of $g$ are the $x$-intercepts of the graph of $g$.

   **b.** Set $g(x)$ equal to 0 and solve for $x$.

**T5.** To see the relationship between the zeros of $g$ and the value of $x$ for which $g$ takes on its minimum value, place your finger at the vertex of $g$'s graph and move straight up to the $x$-axis. Where is your finger in relation to the zeros of $g$?

**S5.** Halfway between the zeros of $g$

**T6.** So what is the numerical relationship between the zeros of $g$ and the value of $x$ for which $g$ takes on its minimum value?

**S6.** The $x$-value is the average of the zeros.

**T7.** To test your conclusion, let's consider another quadratic function, such as $h(x) = -x^2 + 4x$. Before graphing $h$, tell me:

   **a.** What are the zeros of $h$? (*Hint:* Solve the equation $-x^2 + 4x = 0$ by factoring.)

   **b.** What will be the value of $x$ for which $h$ has a minimum or a maximum value?

   **c.** What will be the minimum or maximum value of $h$? (*Hint:* It's the value of $h$ at 2.)

**S7.** **a.** The zeros of $h$ are 0 and 4.

   **b.** The $x$-value will be 2, because that's the average of 0 and 4.

   **c.** $h(2) = -2^2 + 4(2) = 4$

**T8.** Leaving the graph of $g(x) = x^2 + 4x$ on the screen, confirm your predictions about $h$ by graphing $y = -x^2 + 4x$. (The graph of $h$ will appear dotted.)

**S8.** [See diagram.]

**T9.** Notice that $g(x) = x^2 + 4x$ has a minimum value (because its graph opens upward) and that $h(x) = -x^2 + 4x$ has a maximum value (because its graph opens downward). Both functions have the form $f(x) = ax^2 + 4x$. How does the quadratic coefficient $a$ determine whether $f$ has a minimum or a maximum value?

**S9.** If $a$ is positive, $f$ has a minimum value; if $a$ is negative, $f$ has a maximum value.
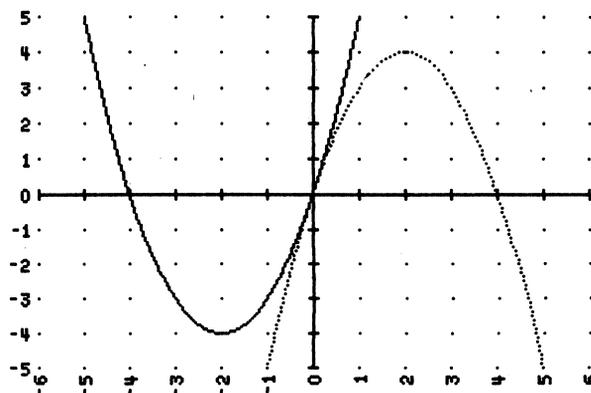
**T10.** Let's look at a slightly more complicated quadratic function, say
$j(x) = 2x^2 - 5x$. Before graphing $j$, tell me:

   **a.** What are the zeros of $j$?

   **b.** What will be the value of $x$ for which $j$ has a minimum or a maximum value?

   **c.** What will be the minimum or maximum value of $j$?

   **d.** Will this value be a minimum or a maximum? Why?

**S10. a.** The zeros of $j$ are 0 and $\frac{5}{2}$ (or 2.5).

   **b.** The $x$-value will be $\frac{5}{4}$ (or 1.25), because that's the average of

     0 and $\frac{5}{2}$.

   **c.** $j\left(\frac{5}{4}\right) = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) = -\frac{25}{8}$ (or $-3.125$)

   **d.** A minimum, because the quadratic coefficient is positive.

**T11.** Confirm your predictions by graphing $y = 2x^2 - 5x$.

**S11.** [See diagram.]

**T12.** Along with the graph of $j$, graph the
functions $k(x) = 2x^2 - 5x + 2$ and
$l(x) = 2x^2 - 5x - 3$. (Before graphing,
change the scale so that the graph displays
the intervals $-10 \le x \le 10$ and $-8 \le y \le 8$.
The graph of $k$ will be dotted; the graph of $l$
will be dashed.)
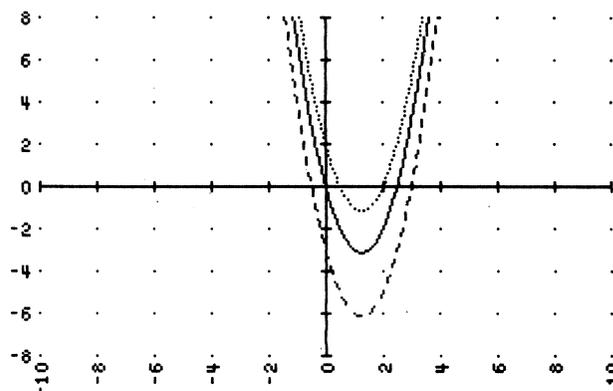
**S12.** [See diagram again.]

**T13.** The three functions that you've graphed all
have the form $f(x) = 2x^2 - 5x + c$ where $c$ is
a constant (in this case 0, 2, and $-3$).
What effect does $c$ have on the graphs?

**S13.** The graphs are the same except for a vertical
shift.

**T14.** Since the graphs are moved vertically (but not horizontally), we would
expect the zeros of each function to be different, which they are. But what
can we say about the value of $x$ for which each function has a minimum or
a maximum value?

**S14.** The $x$-value remains the same.

**T15.** Let's generalize: Given a quadratic function $f(x) = ax^2 + bx + c$, which of
the coefficients $a$, $b$, and $c$ determine the value of $x$ for which the function
has a minimum or a maximum value?

**S15.** $a$ and $b$

**T16.** So let's try to find the $x$-value in terms of $a$ and $b$. First, ignore $c$ and find
the zeros of $g(x) = ax^2 + bx$. What are they? (*Hint:* Solve $ax^2 + bx = 0$ by
factoring.)

**S16.** The zeros of $g$ are 0 and $-\frac{b}{a}$.

**T17.** What, then, is the value of $x$ for which $g$ has a minimum or a maximum value?

**S17.** The $x$-value is the average of 0 and $-\dfrac{b}{a}$, or $-\dfrac{b}{2a}$.

**T18.** You have just stated a formula for determining the value of $x$ for which the general quadratic function $f(x) = ax^2 + bx + c$ has a minimum or a maximum value: $x = -\dfrac{b}{2a}$. To test this formula, use it on the function $k(x) = -3x^2 + 12x - 5$.

**S18.** $x = -\dfrac{12}{2(-3)} = 2$

**T19.** Confirm your prediction by graphing $y = -3x^2 + 12x - 5$.

**S19.** [See diagram.]

**T20.** Let's summarize what we've discussed. We've considered quadratic functions having the form $f(x) = ax^2 + bx + c$. Without graphing:

    **a.** What can you say about the shape of the graph of $f$?

    **b.** How can you determine the value of $x$ for which $f$ has a minimum or a maximum value?

    **c.** How, then, do you find the minimum or maximum value?

    **d.** And how do you know whether this value is a minimum or a maximum?

**S20.** **a.** It will be a parabola.

    **b.** Use the formula $x = -\dfrac{b}{2a}$.

    **c.** Evaluate the function at $-\dfrac{b}{2a}$.

    **d.** It is a minimum if $a > 0$ and a maximum if $a < 0$.
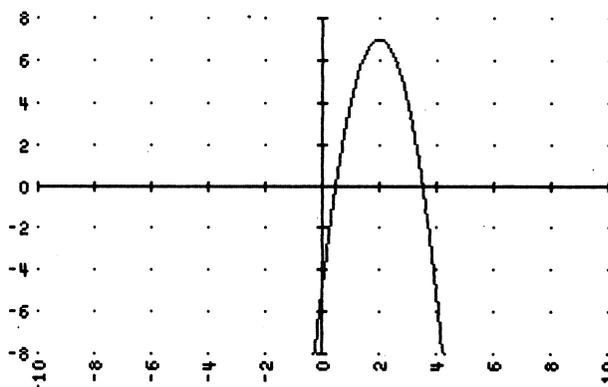
# Enrichment Topics

### Book 1, Lesson 8-9 • Determining Direct Variation

The graph of the direct variation $y = kx$ is a line with slope $k$ that passes through the origin. Students can use this fact to determine whether or not each set of data pairs below represents a direct variation.

Using the Statistics Spreadsheet program, students should plot the data pairs and have the line of best fit drawn.

Select Statistics Spreadsheet. Select ENTER DATA. Relabel "Column A" as "X" and "Column B" as "Y." Enter the data. Select GRAPH. Draw the SCATTER plot, which will include the line of best fit.

If the line passes through all the plotted points as well as the origin, then there is direct variation. When direct variation occurs, have the students state the constant of variation; when it does not, have the students explain why.

Press the space bar to see the equation of the line of best fit. If there is direct variation, the slope of the line of best fit is the constant of variation.

**1.**

| x | y |
|---|---|
| 2 | 3.2 |
| 3 | 5.1 |
| 5 | 7.1 |
| 8 | 13.2 |
| 10 | 16.8 |

[No; the line does not pass through the plotted points.]

**2.**

| x | y |
|---|---|
| 1 | 2.9 |
| 4 | 9.8 |
| 5 | 12.1 |
| 7 | 16.7 |
| 11 | 25.9 |

[No; the line does not pass through the origin.]

**3.**

| x | y |
|---|---|
| 2 | 2.6 |
| 4 | 5.2 |
| 7 | 9.1 |
| 8 | 10.4 |
| 11 | 14.3 |

[Yes; $k = 1.3$]

**4.**

| x | y |
|---|---|
| 3 | 7.2 |
| 4 | 10.1 |
| 6 | 13.7 |
| 9 | 22.6 |
| 10 | 25.3 |

[No; the line does not pass through the plotted points or the origin.]

**Book 1, Lesson 9-4 • The Addition-or-Subtraction Method**

By completing the following exercises using the Line Plotter program, students should gain a better understanding of the addition-or-subtraction method for solving systems of linear equations in two variables. These exercises also serve as an introduction to Lesson 9-5, where multiplication is used with the addition-or-subtraction method.

1. a. Have the students graph the following equations on the same set of axes.

   $x + y = 4$

   $-x + y = 2$

   Ask students to determine from the graph the coordinates of the point of intersection. [(1, 3)]

   b. Have the students add the equations from part (a). The resulting equation [$2y = 6$] should then be graphed along with the original equations. Ask students what they notice about the new line. [It is a horizontal line, and passes through the point of intersection of the original lines.]

   c. Repeat part (b), but this time have the students subtract the equations from part (a). [The resulting line, $2x = 2$, is vertical and passes through the point of intersection of the original lines.]

2. a. Have the students graph the equation $x - y = 7$. Then have the students multiply the equation by 2. [$2x - 2y = 14$] Ask the students what effect they think multiplying by 2 will have on the graph of the equation. [None; it will be the same line.] Then have the students graph both equations to verify their answer.

   b. Have the students graph the equation $-x + 3y = -2$. Then have the students multiply the equation by $-2$. [$2x - 6y = 4$] Ask the students what effect they think multiplying by $-2$ will have on the graph of the equation. [None; it will be the same line.] Then have the students graph both equations to verify their answer.

3. a. Have the students graph the following equations on the same set of axes.

   $2x + y = 3$

   $-x + 2y = 1$

   Ask the students to determine from the graph the coordinates of the point of intersection. [(1, 1)]

   b. Have the students add the equations in part (a). The resulting equation [$x + 3y = 4$] should then be graphed along with the original equations. Ask students what they notice about the new line. [It is slanted and passes through the point of intersection of the original lines.]

   c. Repeat part (b), but this time have the students subtract the equations in part (a). [The resulting line, $3x - y = 2$, is slanted and passes through the point of intersection of the original lines.]

   d. Have students multiply the second equation from part (a) by 2. The system becomes:

   $2x + y = 3$

   $-2x + 4y = 2$

   Ask students where they expect these lines to intersect. [(1, 1)]

e. Have the students add the equations from part (d). The resulting equation [$5y = 5$] should be graphed along with the equations from part (d). Ask students what they notice about the new line. [It is horizontal and passes through the point of intersection of the original lines.]

f. Have students return to the equations in part (a). Ask students by what constant the first equation should be multiplied so that when the equations are added, the graph of the resulting equation will be a vertical line passing through the intersection. [Multiplying the first equation by $-2$ before adding it to the second equation produces $-5x = -5$, whose graph is a vertical line passing through (1, 1).]

## Book 1, Lesson 10-8 • Real-World Applications of Linear Inequalities

To show how systems of linear inequalities can be used in the real world, have students solve the following problems using the Inequality Plotter program.

To keep the graphs as uncluttered as possible, tell the students to graph $x \geq 0$ and $y \geq 0$ mentally whenever these inequalities occur in a problem.

1. A company plans to hire support staff consisting of secretaries and data processors. At least 2 secretaries and at least 4 data processors are needed. Each secretary will be paid $400 per week, and each data processor will be paid $350 per week. The company's budget allows for no more than $3000 per week to be spent on support staff pay.

   a. Let $x$ represent the number of secretaries hired, and let $y$ represent the number of data processors hired. Write a system of three inequalities based on the information given above. [$x \geq 2$, $y \geq 4$, $400x + 350y \leq 3000$]

   b. Graph the system from part (a) on a computer. (Set the scale so that you see only the first quadrant.)

   c. Use the graph to list all possible $x$-$y$ pairs that satisfy the system. Note that $x$ and $y$ must be integers. [$(x, y) = (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (4, 4)$]

2. A student council is sponsoring a dance in the school gym. To pay for the band and decorations for the gym, the student council must raise at least $600 from the sale of admission tickets. A ticket for a couple will be sold for $5, while a ticket for an individual will be sold for $3. Because of fire regulations, no more than 300 people can be admitted to the dance.

   a. Let $x$ represent the number of couples who attend the dance, and let $y$ represent the number of individuals who attend. Write a system of four inequalities based on the information given above. (*Hint:* One inequality is $x \geq 0$. Also bear in mind that a couple represents 2 people.) [$x \geq 0$, $y \geq 0$, $2x + y \leq 300$, $5x + 3y \geq 600$]

   b. Graph the system from part (a) on a computer. (Set the scale so that you see only the first quadrant.)

   c. Describe the region from part (b) that represents both profitability and safety. [It is a quadrilateral with vertices at (150, 0), (120, 0), (0, 200), and (0, 300).]

3. In Problem 2, suppose the student council decides to increase the cost of the dance by spending $.75 per person on refreshments. This is done *without* increasing ticket prices.

   a. How does this decision affect the inequalities from part (a) of Problem 2? [The inequality $5x + 3y \geq 600$ becomes $5x + 3y - 0.75(2x + y) \geq 600$, or $3.5x + 2.25y \geq 600$. All other inequalities are unaffected.]

   b. Graph the revised system from part (a) on a computer.

   c. Use the graph to determine the *maximum* number of couples who can attend the dance without its becoming unprofitable or unsafe. [At most 75 couples can attend.]

4. In Problem 2, suppose the student council decides to increase the cost of the dance by spending $1.00 per person on refreshments. Explain why this should not be done unless ticket prices are raised. [If ticket prices are *not* raised, the region of profitability and safety becomes a single point, (0, 300), which means that 300 individuals (and no couples) *must* attend. Since this is an unrealistic expectation, ticket prices need to be raised.]

## Book 1, Lesson 11-4 • Finding Irrational Square Roots by Graphing

You can have students approximate the square root of a positive number $a$ by graphing the parabola $y = x^2 - a$. For example, have the students use the Function Plotter program to graph $y = x^2 - 2$ on a computer.
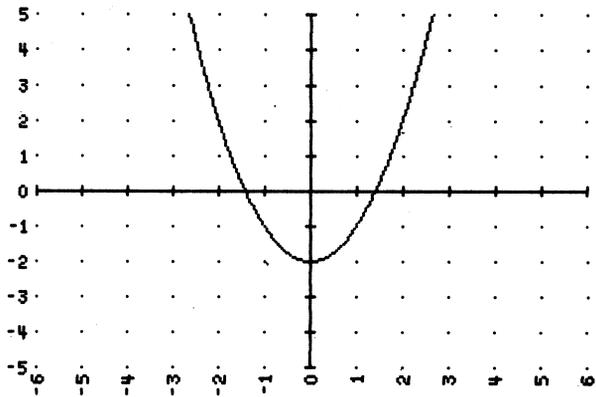


By zooming in on the point where the parabola crosses the positive x-axis, the students can obtain approximations of the point's x-coordinate, which is $\sqrt{2}$. In the diagram at the right, the approximation is 1.4142135.

Likewise, the students can approximate $\sqrt{3}$ by zooming in on the point where the parabola $y = x^2 - 3$ crosses the positive x-axis. You might ask students to explain why this method works. [On the x-axis, $y = 0$, so the equation becomes $0 = x^2 - a$. The roots of this equation are $\pm\sqrt{a}$.]
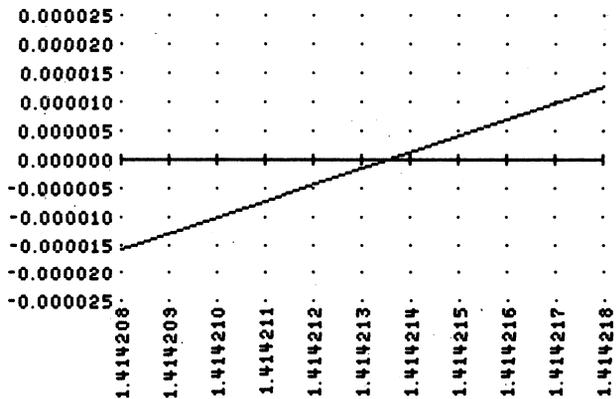
### Book 1, Lesson 11-10 • Solving Simple Radical Equations by Graphing

One method for solving an equation of the form $f(x) = g(x)$ involves graphing the equations $y = f(x)$ and $y = g(x)$ on the same set of axes and then determining the $x$-coordinate of each point where the two curves intersect. Students can use the Function Plotter program and its ZOOM feature to do this.

For example, have the students solve the radical equation $\sqrt{x + 2} = 4 - x$ by graphing $y = \sqrt{x + 2}$ and $y = 4 - x$. (Note that the expression $\sqrt{x + 2}$ is typically entered as SQR(X + 2).) Ask the students for the $x$-coordinate of the point of intersection of the two curves. [2] Then have them substitute this value into the equation $\sqrt{x + 2} = 4 - x$ to confirm that it is a solution. (To provide more of a challenge, ask the students to solve $\sqrt{x + 2} = 5 - x$ by graphing. In this case the $x$-coordinate of the point of intersection is irrational $\left(\dfrac{11 - \sqrt{29}}{2}\right)$, so the students must zoom in to get a reasonable approximation. [2.8])

An alternate approach to solving an equation of the form $f(x) = g(x)$ involves rewriting the equation as $f(x) - g(x) = 0$. When $y = f(x) - g(x)$ is graphed, any $x$-intercepts will be solutions of the equation. For example, the students can solve $\sqrt{x + 2} = 4 - x$ by graphing $y = \sqrt{x + 2} + x - 4$ and determining the $x$-coordinates of the points where the graph crosses the $x$-axis.

### Book 1, Lesson 12-1 • Quadratic Equations with Perfect Squares

Students can use the Parabola Plotter program to solve Written Exercises 1–39 on page 563 by graphing parabolas and then determining their $x$-intercepts.

For example, to solve Exercise 16, students should rewrite the equation $(x + 6)^2 = 16$ as $(x + 6)^2 - 16 = 0$, graph the parabola $y = (x + 6)^2 - 16$, and then zoom in on the point(s) where the parabola crosses the $x$-axis (that is, where $y = 0$).

Although the method of Lesson 12-1 produces exact solutions (many of which are irrational), the graphing method described here produces decimal approximations. Students can use a calculator to compute the decimal approximations of their exact answers and compare them with the answers obtained by graphing.

### Book 2, Lesson 3-2 • Diophantine Equations

Once students are familiar with the fact that the graph of $Ax + By = C$ is a line, they can solve linear Diophantine equations geometrically. (A linear Diophantine equation has the form $Ax + By = C$ where the constants $A$, $B$, and $C$ are integers and where the variables $x$ and $y$ represent integers, usually nonnegative integers.) The problems on the next page involve linear Diophantine equations (as do Problems 1–8 on pages 105–106).

Students may use the Line Plotter program to solve each problem by graphing the line associated with the problem and then determining, from the graph or from a table of values, the coordinates of the first-quadrant lattice points through which the line passes. (A lattice point is a point with integral coordinates.)

1. The perimeter of an isosceles triangle is 10 inches. Find all integral possibilities for the lengths of the sides in inches. Remember that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. [Let $x$ = length of each leg, and let $y$ = length of the base; $2x + y = 10$; $(x, y) = (3, 4), (4, 2)$]

2. Michiko has $.55 in nickels and dimes in her purse. Find all possibilities for the number of each type of coin she could have. Include the possibility of 0 dimes. [Let $x$ = number of nickels, and let $y$ = number of dimes; $5x + 10y = 55$; $(x, y) = (1, 5), (3, 4), (5, 3), (7, 2), (9, 1), (11, 0)$]

3. Fred bought $94 worth of cassette tapes and compact discs at a music store. If the store sells a cassette tape for $8 and a compact disc for $12, find all possibilities for the number of each Fred could have bought. [Let $x$ = number of cassette tapes, and let $y$ = number of compact discs; $8x + 12y = 94$; no solution]

## Book 2, Lesson 3-8 • Determining the Domain and Range of Functions by Graphing

Have students use the Function Plotter program to determine the domain and range of each function given. Students will need to mentally project all graphed points onto the $x$-axis to obtain the domain and onto the $y$-axis to obtain the range. If a graph extends beyond the limits of the computer's screen, students should rescale until they are sure of the extent of the graph.

1. $y = \sqrt{9 - x^2}$

   [$D = \{x: -3 \le x \le 3\}, R = \{y: 0 \le y \le 3\}$]

2. $y = x^2 + 1$

   [$D = \{\text{real numbers}\}, R = \{y: y \ge 1\}$]

3. $y = \dfrac{2x}{x^2 + 1}$

   [$D = \{\text{real numbers}\}, R = \{y: -1 \le y \le 1\}$]

4. $y = \dfrac{x}{|x|}$ $\left\{\begin{array}{l}\text{Students may need to be reminded that software}\\\text{is imperfect: The graph will not show that this}\\\text{function is undefined at zero.}\end{array}\right.$

   [$D = \{x: x \ne 0\}, R = \{1, -1\}$]

## Book 2, Lesson 3-10 • Solving Absolute Value Inequalities

The graphs of all equations having the form $y = a\,|bx + c| + d$ are V-shaped curves. Students can use such graphs, produced by a computer, to solve absolute-value equations and inequalities in a single variable.

For example, to solve $|x + 3| - 1 = 0$, students could use the Absolute Value Plotter to graph the equation $y = |x + 3| - 1$ and then determine the values of $x$ for which the graph crosses the $x$-axis (that is, where $y = 0$).

Similarly, to solve $|2x - 1| < 3$, students should rewrite the inequality as $|2x - 1| - 3 < 0$, graph the equation $y = |2x - 1| - 3$, and then determine the values of $x$ for which the graph lies below the $x$-axis (that is, where $y < 0$). Use this method to solve the A- or B-level Written Exercises for Lessons 2-4 and 2-5.

## Book 2, Lesson 4-7 • Solving Polynomial Equations by Graphing

Have students use the Function Plotter program to check their solutions of Written Exercises 1–50 on pages 196–197. For example, to find the solutions of $6x^2 = 1 - x$ (Exercise 14), students should rewrite the equation as $6x^2 + x - 1 = 0$, graph the equation $y = 6x^2 + x - 1$, and then determine the values of $x$ at which the graph intersects the $x$-axis (that is, where $y = 0$).

You might point out to students what happens when an equation has a root that occurs more than once. If the multiplicity of a root (the number of times the root occurs) is odd, the graph *crosses* the $x$-axis at that root. But if the multiplicity of a root is even, the graph is *tangent* to the $x$-axis at that root.

## Book 2, Lesson 6-1 • Graphing to Find *n*th Roots of Real Numbers

You can have students approximate the $n$th root of a number $a$ by using the Function Plotter program to graph $y = x^n - a$. For example, have students graph $y = x^3 + 4$. By zooming in on the point where the graph crosses the $x$-axis, students can obtain approximations of the point's $x$-coordinate, which is $\sqrt[3]{-4}$. In the diagram at the right below, the approximation is $-1.58740$.

You might wish to point out (or have students discover) that when $n$ is even, the graph of $y = x^n - a$ (where $a$ is a positive real number) crosses the $x$-axis at two points (producing a positive and a negative $n$th root), and when $n$ is odd, the graph of $y = x^n - a$ (where $a$ is any real number) crosses the $x$-axis at only one point (producing a single $n$th root).
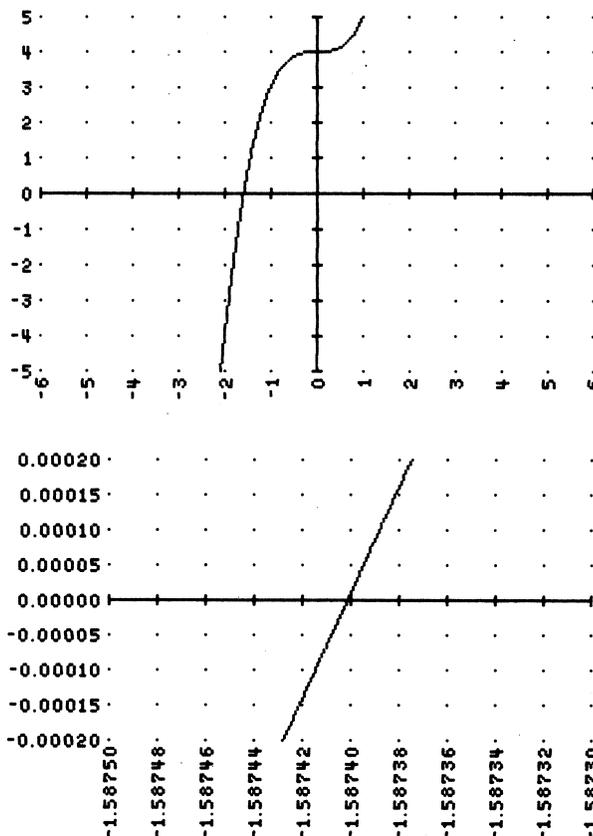
**Book 2, Lesson 8-1 • Curve Fitting**

As a means of introducing students to the concept of curve fitting, have the students use the Statistics Spreadsheet program to determine the relationship between $x$ and $y$ for each data set below. Each set involves a direct variation of the form $y = k \cdot f(x)$ where $k$ is the constant of variation and $f$ is one of the following functions: $f(x) = x, f(x) = x^2, f(x) = \sqrt{x}$.

To determine which function $f$ applies to a given set, students should separately plot the points $(x, y)$, $(x^2, y)$, and $(\sqrt{x}, y)$. On only one of these graphs will the plotted points lie on a straight line that passes through the origin. This identifies the function of $x$ to which $y$ is directly proportional, and the slope of the line is the constant of proportionality. (For example, if the points $(x^2, y)$ lie on a line of slope 2, then the relationship between $x$ and $y$ is given by $y = 2x^2$.)

Select Statistics Spreadsheet. Select ENTER DATA. Relabel "Column A" as "X," "Column B" as "X ^ 2," "Column C" as "SQR(X)," and "Column D" as "Y."

Enter the data for an exercise. (Students will need to calculate the values for the columns $x^2$ and $\sqrt{x}$.)

Select GRAPH. Draw the SCATTER plots for the $x$-$y$, $x^2$-$y$, and $\sqrt{x}$-$y$ pairs.

When the line of best fit that is drawn with each scatter plot passes through all the plotted points (as well as the origin), you have direct variation.

Press the space bar to see an equation of the line of best fit.

For each of the following data sets, find an equation relating $x$ and $y$.

**1.**

| $x$ | $y$ |
|---|---|
| 1 | 0.5 |
| 4 | 8 |
| 9 | 40.5 |

$[y = 0.5x^2]$

**2.**

| $x$ | $y$ |
|---|---|
| 1 | 1.6 |
| 4 | 6.4 |
| 9 | 14.4 |

$[y = 1.6x]$

**3.**

| $x$ | $y$ |
|---|---|
| 1 | 2.4 |
| 4 | 4.8 |
| 9 | 7.2 |

$[y = 2.4\sqrt{x}]$

**4.**

| $x$ | $y$ |
|---|---|
| 1 | 3.7 |
| 4 | 14.8 |
| 9 | 33.3 |

$[y = 3.7x]$

**Book 2, Lesson 8-8 • Approximations with Greater Accuracy**

Students can use the Function Plotter program to approximate the real zeros of the polynomial functions given in Written Exercises 1–10 on page 388 to a higher degree of accuracy than called for in the directions (for example, to the nearest thousandth). [See the answers on the next page.]

Select Function Plotter. Select EQUATIONS and enter the function. DRAW the graph. Select ZOOM and choose one of the intersections of the graph with the $x$-axis.

By repeated ZOOMing, you can obtain better approximations of the root.

To approximate another root, select SCALE and set it to STANDARD SCALE. Then choose a different intersection of the graph with the $x$-axis and repeat the ZOOM procedure.

[Answers: **1.** 2.520  **2.** –2.224  **3.** –1.466  **4.** 1.670  **5.** 2.130
**6.** –0.442  **7.** –1.532, –0.347, 1.879  **8.** –2.115, 0.254, 1.861
**9.** 1.491, 2.763  **10.** –3.359, –1.242, 0.881, 2.720]

## Book 2, Lesson 9-3 • Exploring Parabolas

In the gray summary box on page 414, students are told that the parabola
$y - k = a(x - h)^2$ opens upward if $a > 0$ and downward if $a < 0$.

Have students use the Parabola Plotter program to investigate more thoroughly
the influence of $a$ on the graph of $y - k = a(x - h)^2$. That is, students should hold
$h$ and $k$ constant (say, $h = k = 0$) and vary $a$ to see how the value of $a$ affects the
graph. Students should then write their observations in a clear and concise form.
[The closer $|a|$ is to 0, the flatter the parabola becomes.]

You might also have students use the fact that $a = \frac{1}{4c}$ to deduce from their
observations of $a$ how the varying of $c$ (which controls the distance between a
parabola's focus and directrix) affects the graph. That is, students should be able
to explain how increasing or decreasing the distance between a parabola's focus
and directrix changes the parabola. [The farther apart a parabola's focus and
directrix are, the flatter the parabola becomes.]

## Book 2, Lesson 10-3 • Functions with Inverses on a Restricted Domain

To make students more familiar with functions that do not have inverse
functions, have students use the Function Plotter program to graph each of the
functions given below.

When entering functions, students may use X^2 for $x^2$, ABS(X) for $|x|$, and
SQR(X) for $\sqrt{x}$.

Although none of the graphs satisfies the horizontal-line test, some *portion* of
each graph does. For each graph, then, have students identify a portion that
satisfies the horizontal-line test by stating the interval on the $x$-axis over which
the portion lies. In other words, have students determine a *restricted* domain on
which each function has an inverse function. [Answers may vary.]

**1.** $y = (x + 1)^2$     $[\{x: x \le -1\}$ or $\{x: x \ge -1\}]$

**2.** $y = |2x - 1|$     $[\{x: x \le \frac{1}{2}\}$ or $\{x: x \ge \frac{1}{2}\}]$

**3.** $y = \sqrt{x^2 + 4}$     $[\{x: x \le 0\}$ or $\{x: x \ge 0\}]$

**4.** $y = \dfrac{4}{x^2 + 1}$     $[\{x: x \le 0\}$ or $\{x: x \ge 0\}]$

## Book 2, Lesson 10-4 • Graphing Logarithmic Equations

As a preview of the change-of-base formula that will be presented in
Lesson 10-6, have students use the Function Plotter program to graph the
common log function $y = \log x$ (whose base, though unwritten, is understood to
be 10) and the horizontal line $y = 1$. (When entering functions, use LOG(X) for
$\log x$.)

Ask students to explain why the $x$-coordinate of the point of intersection of the
two graphs is the base of the logarithmic function. [The graphs of $y = \log x$ and
$y = 1$ intersect where $\log_b x = 1$, that is, where $x = b^1 = b$.]

Point out that each equation below, though it involves the common log function, in fact represents a logarithmic function of some base other than 10. Graphs of the three equations are shown at the right. The scale was set to display the interval $0 \leq x \leq 12$.

Have students rewrite each equation in the form $y = \log_b x$ after determining the value of $b$. One way to determine $b$ is to graph each given equation along with $y = 1$ and then find the $x$-coordinate of the point of intersection.
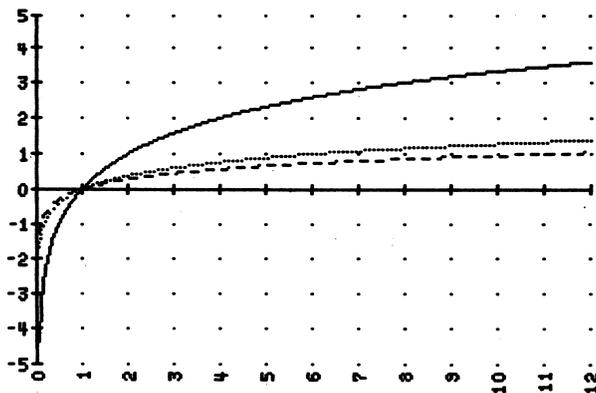


1. $y = \dfrac{\log x}{\log 2}$    2. $y = \dfrac{\log x}{\log 6}$    3. $y = \dfrac{\log x}{\log 11}$

   $[y = \log_2 x]$        $[y = \log_6 x]$        $[y = \log_{11} x]$

Finally, ask students what general conclusion may be drawn. $\left[\log_b x = \dfrac{\log x}{\log b}\right]$

## Book 2, Lesson 10-4 • Curve Fitting Using Exponential Functions

To give students experience with fitting a curve to data, have students use the Statistics Spreadsheet program to determine an equation that best fits the data given in Exercise 1 of "Application–Population Growth Curves" on page 213 of the *Resource Book*. Students should take the following steps:

1. Let $x$ represent the number of years since 1800, and let $y$ represent the base 10 logarithm of the population in millions. Convert the year-versus-population table given in part (a) of Exercise 1 to an $x$-$y$ table.
   $[(x, y) = (0, 0.699), (50, 1.362), (100, 1.881), (150, 2.179)]$

2. Use a computer to plot the $x$-$y$ pairs from Exercise 1 and to determine an equation of the line of best fit. $[y = 0.010x + 0.786]$

3. Into the equation from Exercise 2, substitute $t$ for $x$ and $\log P$ (where $P$ represents the population in millions) for $y$. Then solve for $P$ by rewriting the equation in exponential form and simplifying. $[P = 6.110(1.023)^t]$

## Book 2, Lesson 13-4 • Graphing the Taylor Series for Sine

Once students are familiar with the sine curve, have them look again at the Historical Note on page 573. Students should use the Function Plotter program to compare the graph of $y = \sin x$ with the graph of each equation listed on the next page. In the diagram at the right, the solid curve represents the sine curve, the dotted curve is the graph of Equation 3, and the the dashed curve is the graph of Equation 4.
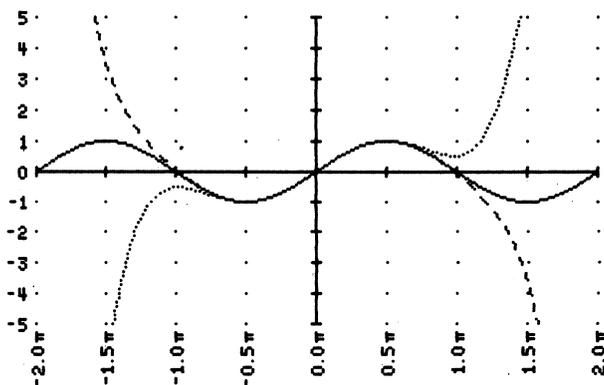


To enter the equations, use ^ for exponentiation and multiplication for the factorials, in this way:
Y = X – (X ^ 3) / (3 * 2) + (X ^ 5) / (5 * 4 * 3 * 2).
Before drawing the equations, select SCALE and set it to show multiples of pi.

Note that each equation includes one more term of the Taylor series for sin $x$ than the equation before it. Students should observe that the graph of each partial Taylor series fits the sine curve well only on a small $x$-interval centered at the origin. Ask students to state this interval of best fit for each partial Taylor series. [Answers may vary. Reasonable intervals are given.]

1. $y = x$ $\qquad$ $[\{x: -\frac{\pi}{6} \le x \le \frac{\pi}{6}\}]$

2. $y = x - \frac{x^3}{3!}$ $\qquad$ $[\{x: -\frac{\pi}{3} \le x \le \frac{\pi}{3}\}]$

3. $y = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ $\qquad$ $[\{x: -\frac{\pi}{2} \le x \le \frac{\pi}{2}\}]$

4. $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ $\qquad$ $[\{x: -\pi \le x \le \pi\}]$

5. Ask students to explain why *no* polynomial function (no matter how many terms it has) can approximate the sine function well for all $x$-values.
[The values of a polynomial function eventually become larger and larger (in absolute value), whereas the values of the sine function always stay between $-1$ and $1$, inclusive.]

## Book 2, Lesson 13-9 • Verifying Trigonometric Identities by Graphing

You and your students can use the Function Plotter program to verify trigonometric identities visually. For example, to show that sine and cosine are cofunctions, you can graph the equations $y = \sin x$ and $y = \cos\left(\frac{\pi}{2} - x\right)$ together and point out that the two graphs coincide. When entering equations, use parentheses around the arguments of trigonometric functions, and type PI for $\pi$.

Some students think that $\sin 2x = 2 \sin x$. To show that this is not the case, you can graph the equations $y = \sin 2x$ and $y = 2 \sin x$ together and point out that the two graphs do not coincide. When using this graphing approach to verify identities, bear in mind that identities involving more than one variable (such as the angle addition formulas) cannot be graphed unless one of the variables is held constant.

Students can also use graphing to check their simplifications of trigonometric expressions (such as Written Exercises 1–30 and 39–48 on pages 639–640, Written Exercises 13–14 and 21–22 on pages 643–644, and Written Exercises 5–12 on page 649).

## Book 2, Lesson 14-8 • Solving Trigonometric Equations by Graphing

When the solutions of a trigonometric equation are to be expressed in radians, students can use the Function Plotter program to solve the equation.

For example, to find the primary solutions of the equation $3 \sec x = \csc x$ (see Exercise 15 on page 700), students can graph the equations $y = 3 \sec x$ and $y = \csc x$ together and determine the $x$-coordinates of all points where the two graphs intersect on the interval $0 \le x < 2\pi$.

An alternate approach would involve rewriting $3 \sec x = \csc x$ as $3 \sec x - \csc x = 0$, graphing the equation $y = 3 \sec x - \csc x$, and then determining all $x$-intercepts of the graph on the interval $0 \le x < 2\pi$.

**Book 2, Lesson 16-6 • Finding the Inverse of a Matrix**

Have students use the Matrix Reducer program to find inverses of matrices. The steps to be followed are:

1. Start with a square matrix $A$.

2. Augment $A$ with an identity matrix of the same size.

3. Perform row operations on the augmented matrix until $A$ becomes an identity matrix. What was formerly an identity matrix is now $A^{-1}$.

For example, to find the inverse of the matrix $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$, perform row operations on the augmented matrix $\begin{bmatrix} 5 & 3 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$ until it becomes $\begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -5 \end{bmatrix}$. The matrix $\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$ is the inverse of the matrix $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$. Students can use this procedure to check their answers for Written Exercises 1–8 on page 792.

Students can also use the Matrix Reducer program to solve systems of equations, such as in Exercises 1–6 on page 786. For example, to solve the system:

$$x + y + z = 0$$
$$x - y + z = 2$$
$$x - y - z = 10$$

students must rewrite the system in matrix form:

$$\begin{matrix} A & X & = & B \end{matrix}$$
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix}$$

The system is solved by finding $A^{-1}$ and then multiplying both sides of the equation by $A^{-1}$ because of the following justification:

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$IX = A^{-1}B$$
$$X = A^{-1}B$$

Students can use the Matrix Reducer program to find $A^{-1}$:

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

The solution of the system is:

$$X = A^{-1}B = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ -4 \end{bmatrix}$$

# Graphs of Linear Equations in Standard Form

Use the Line Plotter program of Algebra Plotter Plus. Select EQUATIONS. Then use the right-arrow key to move to the standard form and fill in the blanks for the missing coefficients. Press the ‹ESC› key and select DRAW.

1. Graph these equations:    $x - y = 1$,    $2x - y = 1$,    $3x - y = 1$

   a. Write the equation whose graph is the steepest: _____

   b. The $y$-coordinate of a point where a graph crosses the $y$-axis is called the $y$-intercept of the graph. What do you notice about the $y$-intercepts of the graphs of the given equations?

   _____

2. Graph these equations:    $x + 2y = 1$,    $x - 2y = 1$

   The equations differ only in the sign of the coefficient of $y$. How do the graphs reflect this difference?

   _____

3. Graph these equations:    $x + y = 2$,    $x + y = -3$,    $x + y = 0$

   The equations have the form $x + y = C$. What effect does changing the number $C$ have on the graph of $x + y = C$?

   _____

4. Graph these equations:    $x + y = 2$,    $x - 2y = 4$,    $x + 4y = 6$

   a. Complete the table below.

   | Equation | $y$-intercept |
   |----------|---------------|
   | $x + y = 2$ | _____ |
   | $x - 2y = 4$ | _____ |
   | $x + 4y = 6$ | _____ |

   b. The equations have the form $x + By = C$. Describe how the numbers $B$ and $C$ determine the $y$-intercept of the graph of $x + By = C$.

   _____

   _____

# *Slope-Intercept Form*

Use the Line Plotter program of Algebra Plotter Plus. You must first select **EQUATIONS** and fill in the blanks for the missing coefficients, then press the ‹ESC› key and select **DRAW**.

1. Graph this equation: $y = 2x$

   Choose any point on the graph. If you move 1 unit to the right of this point, describe how you must move vertically to get back to the line. _____

2. Graph this equation: $y = -3x$

   Choose any point on the graph. If you move 1 unit to the right of this point, describe how you must move vertically to get back to the line. _____

3. Based on the results of Exercises 1 and 2, describe how the line $y = mx$ looks for:
   a. positive values of $m$ _____
   b. negative values of $m$ _____

4. For what value of $m$ would the graph of $y = mx$ be horizontal? _____

5. Graph these equations: $y = \frac{1}{2}x + 3$, $y = \frac{1}{2}x$, $y = \frac{1}{2}x - 3$

   a. How are the three lines geometrically related? _____
   b. What can you say about the slopes of the three lines? _____
   c. What general conclusion do parts (a) and (b) suggest?

   _____

6. Graph these equations: $y = -x + 2$, $y = 3x - 4$, $y = 0x + 1$
   a. Locate the point where each line crosses the y-axis. The y-coordinate of this point is called the *y-intercept* of the line. Complete the table below.

   | Equation | y-intercept |
   |---|---|
   | $y = -x + 2$ | _____ |
   | $y = 3x - 4$ | _____ |
   | $y = 1$ | _____ |

   b. The equations have the form $y = mx + b$. What do you notice about the y-intercepts of the graphs of such equations? _____

7. Try to complete the table below before graphing the given equations.

   | Equation | Slope | y-intercept |
   |---|---|---|
   | $y = -\frac{3}{2}x$ | _____ | _____ |
   | $y = 2x + 1$ | _____ | _____ |
   | $y = -3.5$ | _____ | _____ |

   Now examine the graphs of the given equations to confirm the entries in the table.

# *Zeros of Functions*

**Use the Function Plotter program of Algebra Plotter Plus.**

1. A *zero* of a function is any $x$-value with a corresponding $y$-value of zero. The zeros of a function are where the graph of the function crosses the $x$-axis.

   Graph the following three linear equations and list the zeros you observe.

   | Equations | Zeros |
   |-----------|-------|
   | $y = (x + 5)$ | |
   | $y = (x + 1)$ | |
   | $y = (x - 4)$ | |

2. Modify the equations by using the right-arrow key and introducing another factor. Draw the three quadratic equations and list the zeros you observe.

   | Equations | Zeros |
   |-----------|-------|
   | $y = (x + 5)(x + 4)$ | |
   | $y = (x + 1)(x - 1)$ | |
   | $y = (x - 4)(x - 4)$ | |

3. Modify the equations by introducing yet another factor. Draw the three cubic equations and list the zeros you observe.

   | Equations | Zeros |
   |-----------|-------|
   | $y = (x + 5)(x + 4)(x + 2)$ | |
   | $y = (x + 1)(x - 1)(x - 2)$ | |
   | $y = (x - 4)(x - 4)(x - 4)$ | |

4. Predict the zeros of $y = (x + 3)(x)(x - 4)$. Verify your prediction by graphing the equation. zeros: _____

5. Investigate how multiplying an equation by a nonzero real number influences the zeros of the resulting graph. Enter and draw the graphs of the following equations.

   | Equations | Zeros |
   |-----------|-------|
   | $y = (x - 1)(x + 1)(x + 2)$ | |
   | $y = (2)(x - 1)(x + 1)(x + 2)$ | |
   | $y = (-1.5)(x - 1)(x + 1)(x + 2)$ | |

   Does multiplying an equation by a nonzero real number change the zeros? _____

**Guess the zeros of the functions and check by graphing the equations.**

6. $y = (0.25x + 0.75)(3x - 6)$          7. $y = (0.5x + 1)(2x - 5)$

8. $y = (0.2)(x + 2.25)(x + 0.5)(x)(x - 1.5)(x - 3)$

9. *Challenge:* $y = (0.05)(2x + 6)(2x + 1)(3x - 2)(0.5x - 1)$

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# *Inverse Variation*

Use the Function Plotter program of Algebra Plotter Plus. Set the SCALE
so the graphs display the intervals $-8 \le x \le 8$ and $-8 \le y \le 8$.

1. Graph this equation:     $xy = 4$

    (Solve the equation for $y$ before entering it on the computer.)

    a. This equation is an example of an *inverse variation*. Its graph is called a
    *hyperbola*. Notice that the hyperbola has two branches. In what two
    quadrants do the branches lie? _____

    b. What does the graph tell you about the $y$-values as the *positive* $x$-values
    get larger and larger? _____

    c. What does the graph tell you about the $y$-values as the *negative* $x$-values
    get smaller and smaller? _____

    d. What does the graph tell you about the $y$-values as the *positive* $x$-values
    get closer and closer to 0? _____

    e. What does the graph tell you about the $y$-values as the *negative* $x$-values
    get closer and closer to 0? _____

    f. Is the inverse variation defined for $x = 0$? Explain.

    _____

2. Graph the direct variation $y = 6x$ and the inverse variation $y = \dfrac{6}{x}$.

    Complete the following sentences.

    a. In the direct variation, as the positive $x$-values increase, the
    corresponding $y$-values _____.

    b. In the inverse variation, as the positive $x$-values increase, the
    corresponding $y$-values _____.

3. To find algebraically the solution to the system of equations in Exercise 2,
    you must discover the values of $x$ for which $y = 6x$ and $y = \dfrac{6}{x}$ have the same
    $y$-value. In this exercise, you will find the values of $x$ for which the
    following is true: $6x = y = \dfrac{6}{x}$.

    a. Solve the equation $6x = \dfrac{6}{x}$ to determine the two $x$-values that satisfy both
    equations of the system. $x =$ ___ and $x =$ ___

    b. Use either equation to calculate the $y$-values that are associated with each
    $x$-value you found in part (a). $y =$ ___ and $y =$ ___

    c. Look at the graph you used in Exercise 2. At what two points does the
    hyperbola intersect the line? If necessary, select SCALE and change the
    intervals of the displayed graph to see the intersection points.

    ( ___ , ___ ), ( ___ , ___ )

# Solving Systems
# of Linear Equations

Use the Line Plotter program of Algebra Plotter Plus.  Select EQUATIONS
and use the right-arrow key to move to the standard form.

1.  Graph this system:   $2x + y = 3$
    $\qquad\qquad\qquad\quad x - 3y = 5$

    Write the coordinates of the point of intersection: ( _____ , _____ )

    This is called a *solution* of the system.  Check the solution by substituting
    the $x$- and $y$-coordinates of the point of intersection into each equation of the
    system.  You should get a true statement each time.

2.  Graph this system:   $2x + 2y = 1$
    $\qquad\qquad\qquad\quad x - 2y = 4$

    a.  The coordinates of the point of intersection for the system are *not*
        integers.  Estimate the coordinates from the graph: ( _____ , _____ )

    b.  To obtain a better approximation of the solution, select ZOOM and move
        the cross hairs near the point of intersection.  Press ‹RETURN› or
        ‹ENTER› and use the arrow keys to change the size of the rectangle that
        indicates the portion of the graph to be magnified.  Then press
        ‹RETURN› or ‹ENTER› and the computer will automatically draw the
        magnified graph.  Now give a better estimate of the coordinates of the
        point of intersection: ( _____ , _____ )

3.  Graph this system:   $x - y = 1$
    $\qquad\qquad\qquad\quad 3x + 4y = 0$

    Find the solution of the system to the nearest tenth: ( _____ , _____ )
    (*Note:* If you used ZOOM in the previous exercise, you may first need to
    return the scale to the standard values.  Select SCALE and use arrow keys to
    move to the "Standard Scale" option, then highlight "Yes.")

4.  Graph this system:   $-2x + y = 13$
    $\qquad\qquad\qquad\quad 5x + 4y = -30$

    Find the solution of the system to the nearest tenth: ( _____ , _____ )
    (*Note:* If the point of intersection is off the screen, you will need to rescale
    to see where the lines intersect.  Select SCALE and redefine the minimum
    and maximum $x$-values before using DRAW.)

5.  Graph this system:   $6x - 4y = 3$
    $\qquad\qquad\qquad\quad 9x - 6y = 8$

    a.  What do you notice about the lines? _____
        What can you say about the solution of the system? _____

    b.  By replacing the constant 8 in the second equation with some other
        constant, you can make the graph of the system be two coinciding lines.
        What should the constant be? _____ What can you say about the solution
        of the system when the lines coincide? _____

        _____

# *Absolute Value Graphs*

**Use the Absolute Value Plotter program of Algebra Plotter Plus.**

1. Graph these equations:     $y = |x|$,        $y = |x| + 1$,      $y = |x| - 2$

   a. Each V-shaped graph has a lowest point, called the *vertex*. Locate the vertex of each graph and complete the table below.

   | Equation | Input format | Vertex |
   |----------|--------------|--------|
   | $y = |x|$ | y − (0) = (1)Abs(x − (0)) | (0, 0) |
   | $y = |x| + 1$ | y − (1) = (1)Abs(x − (0)) | ( ___ , ___ ) |
   | $y = |x| - 2$ | y − (−2) = (1)Abs(x − (0)) | ( ___ , ___ ) |

   b. The equations have the form $y = |x| + C$. Using the number $C$, give the coordinates of the vertex of the graph of $y = |x| + C$: ( ___ , ___ )

2. Graph these equations:     $y = |x|$,        $y = |x - 1|$,      $y = |x + 2|$

   a. Complete the table below.

   | Equation | Input format | Vertex |
   |----------|--------------|--------|
   | $y = |x|$ | y − (0) = (1)Abs(x − (0)) | ( ___ , ___ ) |
   | $y = |x - 1|$ | y − (0) = (1)Abs(x − (1)) | ( ___ , ___ ) |
   | $y = |x + 2|$ | y − (0) = (1)Abs(x − (−2)) | ( ___ , ___ ) |

   b. The equations have the form $y = |x + B|$. Using the number $B$, give the coordinates of the vertex of the graph of $y = |x + B|$: ( ___ , ___ )

3. Based on the results of Exercises 1 and 2, try to complete the table below before graphing the given equations.

   | Equation | Input format | Vertex |
   |----------|--------------|--------|
   | $y = |x - 3| - 1$ | y − ( ) = ( )Abs(x − ( )) | ( ___ , ___ ) |
   | $y = |x - 2| + 2$ | y − ( ) = ( )Abs(x − ( )) | ( ___ , ___ ) |
   | $y = |x + 1| - 4$ | y − ( ) = ( )Abs(x − ( )) | ( ___ , ___ ) |

   Now examine the graphs of the given equations to confirm the entries in the table.

4. Find an equation involving absolute value for which the graph of the equation has its vertex at:

   a. (−2, 4) _____     b. (1, −3) _____

   Confirm your equations by graphing.

# *Linear Programming*

**Use the Inequality Plotter program of Algebra Plotter Plus.**

1.  Suppose the feasible region for a linear programming problem is given by:

    $x \geq 0$

    $y \geq 0$

    $x + y \leq 9$

    $x + 2y \leq 16$

    $4x + y \leq 24$

    Since the first two inequalities imply that the feasible region lies entirely in the first quadrant, you need not plot them. Instead, you should set the SCALE to display the intervals $0 \leq x \leq 12$ and $0 \leq y \leq 10$. Then select INEQUALITIES and use the arrow keys to fill in the blanks for the other three inequalities. The region that contains the densest shading is the feasible region. List the five vertices of the feasible region:

    ( ____ , ____ ), ( ____ , ____ ), ( ____ , ____ ), ( ____ , ____ ), ( ____ , ____ )

2.  For the feasible region of Exercise 1, suppose you are given the objective function $P = 3x + 2y$. To determine the minimum or maximum value of $P$ in the feasible region, you need only check the values of $P$ at the vertices.

    For example, one of the vertices in your list for Exercise 1 should have been the origin, as shown in the table below. Complete the table for the other vertices from your list.

|     | $x$ | $y$ | $3x + 2y = P$ |
|-----|-----|-----|---------------|
|     | 0   | 0   | $3(0) + 2(0) = 0$ |
| a.  |     |     |               |
| b.  |     |     |               |
| c.  |     |     |               |
| d.  |     |     |               |

   e.  The minimum value of $P$ is ___, which is found when $x =$ __ and $y =$ __.

   f.  The maximum value of $P$ is ___, which is found when $x =$ __ and $y =$ __.

3.  Use the feasible region of Exercise 1 to find the minimum and maximum values of the objective function $P = 4x + 5y$.

    a.  The minimum value of $P$ is ___, which is found when $x =$ __ and $y =$ __.

    b.  The maximum value of $P$ is ___, which is found when $x =$ __ and $y =$ __.

# *Exploring Slopes*

**Use the Line Plotter program of Algebra Plotter Plus.**

1. Graph these equations:  $y = \frac{1}{2}x,$  $y = 1x,$  $y = 2x$

   The coefficient of $x$ in each equation is called the *slope* of the line. What do you notice about the steepness of the lines as the slopes get larger?

   _____

2. Notice that the lines in Exercise 1 all pass through the origin. Find the point on each line with $x$-coordinate 1 and complete the table below.

   | Equation of line | Slope | Common point | Point with x-coordinate 1 |
   |---|---|---|---|
   | **a.** $y = \frac{1}{2}x$ | _____ | (0, 0) | (1, ___) |
   | **b.** $y = 1x$ | _____ | (0, 0) | (1, ___) |
   | **c.** $y = 2x$ | _____ | (0, 0) | (1, ___) |

   **d.** Describe the relationship between the point with $x$-coordinate 1 and the slope of any line that passes through the origin.

   _____

   _____

3. Graph these equations:  $y = 1x + 1,$  $y = 0x + 1,$  $y = -1x + 1$
   **a.** What happens to the lines as the slope changes from positive to negative?

   _____

   _____

   **b.** What point do the lines have in common? ( ___ , ___ )

4. Try to complete the table below before graphing the given equations.

   | Equation of line | Slope | Common point | Point with x-coordinate 1 |
   |---|---|---|---|
   | **a.** $y = -\frac{1}{2}x + 1$ | _____ | ( ___ , ___ ) | (1, ___) |
   | **b.** $y = -1x + 1$ | _____ | ( ___ , ___ ) | (1, ___) |
   | **c.** $y = -2x + 1$ | _____ | ( ___ , ___ ) | (1, ___) |

   Now examine the graphs of the given equations to confirm the entries in the table.

5. Use the "Line Quiz" program of Algebra Plotter Plus to see if you can correctly determine the equations of random lines drawn by the computer.

NAME _____   DATE _____

# *Parallel and Perpendicular Lines*   | Algebra Plotter Plus | | Book 2: Lesson 3-4 |

**Use the Line Plotter program of Algebra Plotter Plus.**

1. Graph these equations:   $y = 2x + 1,$   $y = 2x,$   $y = 2x - 1$
   a. What geometric relationship do the lines have? _____
   b. What do you notice about the slopes of the lines? _____

2. Graph this equation:   $y = -3x + 1$
   a. Find an equation of the line having the same slope as the given line but passing through the point $(0, -1)$. _____
   b. Compare the graphs of the given equation and your equation from part (a). What do you notice? _____

3. Based on your answers to Exercises 1 and 2, complete the statement:

   Parallel lines have slopes that are _____

4. Graph these equations:   $4x + 3y = 9,$   $8x + 6y = 9$
   a. Compare the graphs. What do you notice? _____
   b. By trial and error, find coefficients $A$, $B$, and $C$ so that the graph of the equation $Ax + By = C$ is parallel to the first equation and passes through the point $(0, -3)$.  $A =$ _____ , $B =$ _____ , $C =$ _____

5. Based on your answers to Exercise 4, complete the following statement:

   The graphs of $Ax + By = C$ and $Dx + Ey = F$ are parallel if and only if

   _____

6. Graph these equations:   $y = \frac{2}{3}x,$   $y = \frac{3}{2}x,$   $y = -\frac{3}{2}x$

   Which of these two lines appear to be perpendicular? _____

   (*Note:* The angle between the perpendicular lines may not be exactly 90° due to distortion of the computer monitor. To reduce the distortion, set the scale so that the graph displays $-6 \le x \le 6$ and $-4 \le y \le 4$.)

7. Graph these equations:   $y = -3x + 1,$   $y = 3x - 2,$   $y = -\frac{1}{3}x$

   Which of these two lines appear to be perpendicular? _____

8. Based on your answers to Exercises 6 and 7, complete the statement:

   Perpendicular lines have slopes that are _____

9. Graph these equations:   $4x + 3y = 9,$   $6x - 8y = 9$
   a. Compare the graphs. What do you notice? _____
   b. By trial and error, find coefficients $A$, $B$, and $C$ so that the graph of the equation $Ax + By = C$ is perpendicular to the first equation and passes through the point $(0, -3)$.  $A =$ _____ , $B =$ _____ , $C =$ _____

10. Based on your answers to Exercise 9, complete the following statement:

   The graphs of $Ax + By = C$ and $Dx + Ey = F$ are perpendicular if and only if

   _____

# Estimating Solutions of Linear Systems

**Use the Line Plotter program of Algebra Plotter Plus. Select EQUATIONS and use the right-arrow key to move to the standard form.**

1. Graph this system:  $3x + 2y = 11$
   $2x - 3y = 3$

   Write the coordinates of the point of intersection: ( ____ , ____ )

   This is called a *solution* of the system. Check the solution by substituting the $x$- and $y$-coordinates of the point of intersection into each equation of the system. You should get a true statement each time.

2. Graph this system:  $8x + y = 2$
   $20x + 5y = 2$

   a. The coordinates of the point of intersection for the system are *not* integers. Estimate the coordinates from the graph: ( _____ , _____ )

   b. To obtain a better approximation of the solution, select ZOOM and move the cross hairs near the point of intersection. Press <RETURN> or <ENTER> and use the arrow keys to change the size of the rectangle that indicates the portion of the graph to be magnified. Then press <RETURN> or <ENTER> and the computer will automatically draw the magnified graph. Now give a better estimate of the coordinates of the point of intersection: ( _____ , _____ )

3. Graph this system:  $2x - 4y = 7$
   $3x + 5y = 9$

   Find the solution of the system to the nearest tenth: ( _____ , _____ )
   (*Note*: If you used ZOOM in the previous exercise, you may first need to return the scale to the standard values. Select SCALE and use arrow keys to move to the "Standard Scale" option, then highlight "Yes.")

4. Graph this system:  $9x - 5y = 13$
   $-7x + 3y = 6$

   Find the solution of the system to the nearest tenth: ( _____ , _____ )
   (*Note*: If the point of intersection is off the screen, you will need to rescale to see where the lines intersect. Select SCALE and redefine the minimum and maximum $x$-values and $y$-values before using DRAW.)

5. Select TABLE to see a table of values for the equations in Exercise 4. Explain how this table helps you find the point of intersection *without* looking at the graph.

   _____

   _____

# *Parabolas*

Use the Parabola Plotter program of Algebra Plotter Plus.  The graphs of
the equations given in the exercises below are all *parabolas*.  Each graph will
have a highest or lowest point, called a *vertex*.  For example, the graph of
$y = x^2$ has a vertex at the origin.

1.  Graph the equations and complete the table below.

| Equation of parabola | Input format | Coordinates of vertex |
|---|---|---|
| $y = -(x + 3)^2$ | y − (0) = (−1)(x − (−3))^2 | ( ____ , ____ ) |
| $y = 3(x - 2)^2$ | y − (0) = (3)(x − (2))^2 | ( ____ , ____ ) |
| $y = -\dfrac{2}{3}(x + 1)^2$ | y − (0) = (−2/3)(x − (−1))^2 | ( ____ , ____ ) |

2.  Graph the equations and complete the table below.

| Equation of parabola | Input format | Coordinates of vertex |
|---|---|---|
| $y - 2 = x^2$ | y − (2) = (1)(x − (0))^2 | ( ____ , ____ ) |
| $y + 1 = -2x^2$ | y − (−1) = (−2)(x − (0))^2 | ( ____ , ____ ) |
| $y - 3 = \dfrac{1}{2}x^2$ | y − (3) = (1/2)(x − (0))^2 | ( ____ , ____ ) |

3.  Try to complete the table below before graphing the given equations.

| Equation of parabola | Input format | Coordinates of vertex |
|---|---|---|
| $y - 2 = (x - 3)^2$ | y − ( ) = ( )(x − ( ))^2 | ( ____ , ____ ) |
| $y - 1 = -3(x + 2)^2$ | y − ( ) = ( )(x − ( ))^2 | ( ____ , ____ ) |
| $y + 3 = \dfrac{2}{3}(x + 1)^2$ | y − ( ) = ( )(x − ( ))^2 | ( ____ , ____ ) |

Now examine the graphs of the given equations to confirm the entries in the
table.

4.  Write equations of three different parabolas with vertex $(2, -1)$.

   a. _____

   b. _____

   c. _____

   Graph the equations to confirm that the parabolas have the correct vertex.

5.  Use the "Parabola Quiz" program of Algebra Plotter Plus to see if you can
   correctly determine the equations of parabolas drawn by the computer.

# Approximating Roots of Polynomial Equations

Use the Function Plotter program of Algebra Plotter Plus. After each exercise, you may wish to reset the boundaries of the graphed region to Standard Scale.

1. Graph this equation:    $y = x^3 - 2x + 3$

   The $x$-coordinate of any point where the graph crosses the $x$-axis is a *root* of the equation $x^3 - 2x + 3 = 0$. As you can see from the graph, the equation has only one real root. To approximate this root, select TABLE to see a table of values. The root must lie between consecutive integral $x$-values for which the corresponding $y$-values have opposite sign. Write down these values:

   |     | $x$ | $y$ |
   |-----|-----|-----|
   | **a.** | _____ | _____ |
   | **b.** | _____ | _____ |

   c. Of the two $x$-values listed above, which is the better approximation of the root? _____

   d. Justify your answer to part (c): _____

   _____

2. Select ZOOM and find an approximation of the root to the nearest hundredth. _____

3. Graph this equation:    $y = x^4 + 3x^2 - x - 4$
   a. How many real roots does the equation $x^4 + 3x^2 - x - 4 = 0$ have? _____
   b. Approximate each root to the nearest hundredth. _____

4. Graph this equation:    $y = 81x^4 - 81x^3 + 9x^2 + 4$

   Notice that the graph is *tangent* to the $x$-axis (that is, the graph touches the $x$-axis without crossing it).

   a. The procedure, described in Exercise 1, for using a table of values to approximate a root does not apply when a graph is tangent to the $x$-axis. Describe an alternate procedure for using a table of values to approximate a root that occurs at a point of tangency:

   _____

   _____

   b. Use your procedure from part (a) to approximate the root of $81x^4 - 81x^3 + 9x^2 + 4 = 0$ to the nearest hundredth: _____

5. Graph this equation:    $y = 147x^3 - 77x^2 - 15x + 9$

   (*Note:* To see the graph better, change the SCALE to display the intervals $-2 \le x \le 2$ and $-10 \le y \le 10$.)

   Approximate all the roots of the equation $147x^3 - 77x^2 - 15x + 9 = 0$ to the nearest hundredth: _____

NAME _____     DATE _____

# *Identifying Central Conics*

Use the Conics Plotter program of Algebra Plotter Plus. To minimize distortion, you may wish to set the SCALE for each exercise so the graph displays the intervals $-6 \le x \le 6$ and $-4 \le y \le 4$.

1. The graph of each of the equations in the table below is a *central conic* (that is, a circle, an ellipse, or a hyperbola). Graph the equations and complete the table. The "center" of a hyperbola is the point of intersection of its asymptotes.

| Equation of conic | Name of conic | Coordinates of center |
|---|---|---|
| $x^2 + 2y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |
| $x^2 - 2y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |
| $x^2 + y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |

2. Repeat Exercise 1 for the equations in the table below.

| Equation of conic | Name of conic | Coordinates of center |
|---|---|---|
| $-1x^2 + -1y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |
| $\frac{1}{2}x^2 + 2y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |
| $-\frac{1}{2}x^2 + 2y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |

3. The equations in the tables of Exercises 1 and 2 all have the form $ax^2 + by^2 - 2x + 4y = -1$. Based on your work so far, make a conjecture about the way in which $a$ and $b$ determine the type of conic that results when an equation of this form is graphed. _____

_____

4. Use Exercise 3 to complete the "Name of conic" column in the table below before graphing the given equations. Then graph the equations and complete the "Coordinates of center" column.

| Equation of conic | Name of conic | Coordinates of center |
|---|---|---|
| $-\frac{1}{2}x^2 + y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |
| $-x^2 - y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |
| $\frac{1}{2}x^2 + y^2 - 2x + 4y = -1$ | | ( ___ , ___ ) |

5. Use the "Conics Quiz" program of Algebra Plotter Plus to see if you can correctly determine the equations of central conics drawn by the computer.

# *Estimating Solutions of Quadratic Systems*

Use the Conics Plotter program of Algebra Plotter Plus. To minimize distortion, you may wish to set the SCALE for each exercise so the graph displays the intervals $-6 \le x \le 6$ and $-4 \le y \le 4$.

1.  Graph these equations:     $x^2 + y^2 = 4$
    $2x + y = 1$

    Notice that the line intersects the circle twice. Write an approximation of the coordinates of each point of intersection: ( ____ , ____ ), ( ____ , ____ )

2.  You can obtain more accurate approximations by using the ZOOM feature. By zooming, approximate the coordinates of Exercise 1 to the nearest hundredth: ( _____ , _____ ), ( _____ , _____ )

3.  Graph these equations:     $x^2 + y^2 = 16$
    $4x^2 + 25y^2 = 100$

    a.  Approximate the coordinates of the point of intersection in the first quadrant to the nearest hundredth: ( _____ , _____ )

    b.  The graph of the given quadratic system is symmetric with respect to the $y$-axis, the origin, and the $x$-axis. Use this fact and your answer to part (a) to write the approximate coordinates of the points of intersection in the second, third, and fourth quadrants:

    ( _____ , _____ ), ( _____ , _____ ), ( _____ , _____ )

4.  Graph these equations:     $x^2 + y^2 + 2x - 4y = 6$
    $x^2 + y^2 + 2x + y = 4$

    a.  Subtract the second equation from the first and write down the result.

    _____

    b.  Graph the equation from part (a). How is this graph related to the graph of the given quadratic system?

    _____

    _____

5.  Graph these equations:     $x^2 + y^2 - 3x + 5y = 8$
    $x^2 + y^2 + 4x + 2y = 3$

    a.  Based on the results of Exercise 4, write an equation of the line that should pass through the points of intersection for the given system.

    _____

    Graph the equation to verify.

    b.  Approximate the coordinates of each point of intersection for the given system to the nearest hundredth:  ( _____ , _____ ), ( _____ , _____ )

# *Exponential Growth*

**Use the Function Plotter program of Algebra Plotter Plus.**

1. Graph these equations:    $y = 2^x$,    $y = 3^x$,    $y = 4^x$

   a. Which graph lies above the other two for $x < 0$? _____

   b. Which graph lies above the other two for $x > 0$? _____

2. Graph these equations:    $y = \left(\frac{2}{3}\right)^x$,        $y = \left(\frac{3}{2}\right)^x$

   a. Which graph is *increasing* from left to right? _____

   b. Which graph is *decreasing* from left to right? _____

3. Based on your answer to Exercise 2, complete the following.

   a. Predict whether the graph of $y = 1.2^x$ is increasing or decreasing. _____

   b. Predict whether the graph of $y = 0.8^x$ is increasing or decreasing. _____
      Now graph the equations to confirm your predictions.

4. Generalize the results of Exercises 2 and 3. Assume $b > 0$.

   a. Under what conditions does the graph of $y = b^x$ increase?

   _____

   b. Under what conditions does the graph of $y = b^x$ decrease?

   _____

   c. When would the graph of $y = b^x$ neither increase nor decrease?

   _____

5. The following table shows the population $y$ of a colony of bacteria over time
   $x$ in hours. Assume the population grows exponentially.

   a. Complete the table by finding $y$ when $x = 4, 5$, and $6$.

   | $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
   |---|---|---|---|---|---|---|---|
   | $y$ | 5 | 10 | 20 | 40 | | | |

   b. Use the pattern in the table to write an equation for $y$ in terms of $x$.

   _____

   c. Find $y$ when $x = 10$: _____ when $x = 12$: _____.

   d. Use the computer to graph the equation in (b). Set the SCALE so that
      the graph will display the interval $0 \le x \le 12$. What interval on the
      $y$-axis must be used to show the function values? _____

   e. Set the SCALE to display the interval $0 \le x \le 4$. From the screen
      display, estimate the population when $x = 2.5$ hours. _____
      Check your estimate by selecting TABLE.

   f. After how many hours was the population one-half of the maximum
      population shown on the screen? _____                           *(continued)*

## *Exponential Growth* (continued)

6. A rare population of tortoises now numbering 38 increases at the rate of 4% per year. Round your answers to the nearest whole tortoise.

    a. Find the population one, two, and three years from now. _____

    b. The equation $y = 38(1.04)^x$ gives the population $y$ after $x$ years. Do your calculations in (a) and (b) agree with this equation? _____

    c. Use the computer to graph the equation in (b) for the interval $0 \le x \le 10$.

    d. Estimate the population to the nearest whole tortoise after 9 years. _____

    e Approximately how many years will it take for the population to reach 51? _____

    f. Approximately how many years will it take for the population to double from 38 to 76? _____

    g. Approximate the number of years it will take for the population to double from 76 to 152. _____ Is your answer the same as (f)? _____

7. Use the computer to graph $y = 10(1.06)^x$. From the screen display, estimate the time it will take for $y$ to change from 10 to 20. _____

8. Repeat Exercise 7 for the function $y = 10(1.04)^x$. _____

9. The growth function of a certain population is $y = 10(1.05)^x$.

    a. Predict the time it takes for the population to double. _____

    b. Use the computer to graph the function in (a) to check your prediction.

**Some populations decline over time. In Exercises 10 and 11, $y$ represents population and $x$ represents time in years.**

10. An initial population of 24 decreases at the rate of 3% per year.

    a. Find the population after 1 year. (*Hint:* If at the start, the population is 100%, then after 1 year, the population is 97% of the original population.) _____

    b. Find the population after two years. That is, find 97% of 97% of 24. _____

    c. Write an equation of the form $y = ab^x$ to describe the population as a function of time. _____

    d. Use the computer to graph the function. Does the screen display indicate that the population is declining? _____

11. An initial population of 40 decreases at the rate of 4% per year.

    a. Write an equation to give the population as a function of time. _____

    b. Use the computer to graph the function over the interval $0 \le x \le 20$.

    c. After how many years will the population be half its original size? _____

    d. Graph the function over the interval $20 \le x \le 40$. Estimate the population after 40 years. _____

# *Functions and Their Inverses*  ⬚ Algebra Plotter Plus ⬚ Book 2: Lesson 10-3

Use the Function Plotter program of Algebra Plotter Plus. To minimize distortion, you may wish to set the SCALE for each exercise so the graph displays the intervals $-6 \le x \le 6$ and $-4 \le y \le 4$.

1. Graph these equations: $\quad y = 3x + 2$

$$y = \frac{(x-2)}{3}$$

$$y = x$$

The functions $f(x) = 3x + 2$ and $g(x) = \frac{(x-2)}{3}$ are *inverses*, since each one "undoes" what the other does. For example, $f(1) = 5$ and $g(5) = 1$. When you compare the graphs of $f$ and $g$, what geometric relationship involving the line $y = x$ do you see?

_____

2. Graph these equations: $\quad y = (x-2)^3$

$$y = \sqrt[3]{x} + 2$$

$$y = x$$

You can enter the new equations by typing: $y = (x-2)^{\wedge}3$ and $y = x^{\wedge}(1/3) + 2$. What do you notice about the graphs of these inverse functions?

_____

3. Graph this equation: $\quad y = \frac{1}{2}x^3$

Based on your answers to Exercises 1 and 2, sketch the graph of the inverse of $f(x) = \frac{1}{2}x^3$. Then graph $y = \sqrt[3]{2x}$ by entering $y = (2x)^{\wedge}(1/3)$ to confirm your sketch.

4. Graph this equation: $\quad y = x^2 - 4$

Sketch the graph of the inverse of $f(x) = x^2 - 4$. Then enter $y = SQR(x + 4)$ and $y = -SQR(x + 4)$ to confirm your sketch. Why do you have to tell the Function Plotter to graph two parts to get the inverse of $f$?

_____

# Sine and Cosine Curves

Use the Function Plotter program of Algebra Plotter Plus. Change the
$x$-axis scale to multiples of $\pi$ by selecting SCALE, moving the cursor to the
"Multiples of pi" option, and selecting "Yes."

1. Graph these equations: $y = \sin(x)$, $y = \cos(x)$
   a. The *period* of each curve is the shortest distance along the $x$-axis over
      which the curve has one complete up-and-down cycle. In terms of $\pi$,
      what is the period of the sine curve? _____ of the cosine curve? _____
   b. The *maximum* value attained by each curve is the $y$-coordinate of the
      highest points on the curve, while the *minimum* value is the $y$-coordinate
      of the lowest points. What are the maximum and minimum values of the
      sine curve? _____ , _____ of the cosine curve? _____ , _____
   c. The *amplitude* of each curve is half the difference between the maximum
      and minimum values of the curve. What is the amplitude of the sine
      curve? _____ of the cosine curve? _____
   d. As you can see from your answers to parts (a)–(c), the sine and cosine
      curves share the same basic features. In fact, if all points on the cosine
      curve were shifted to the right some distance $d$, the translated cosine
      curve would coincide with the sine curve. In terms of $\pi$, what is the
      smallest possible value of $d$? _____

2. Graph these equations: $y = 1 + \sin(x)$, $y = -2 + \cos(x)$

   Using $\pi$ when giving each period, complete the table below.

   | Equation | Period | Maximum | Minimum | Amplitude |
   |---|---|---|---|---|
   | $y = 1 + \sin(x)$ | | | | |
   | $y = -2 + \cos(x)$ | | | | |

3. What effect does the constant $c$ seem to have on the graphs of $y = c + \sin(x)$
   and $y = c + \cos(x)$?

4. Try to complete the table below before graphing the given equations.

   | Equation | Period | Maximum | Minimum | Amplitude |
   |---|---|---|---|---|
   | $y = -3 + \sin(x)$ | | | | |
   | $y = 4 + \cos(x)$ | | | | |

   Now examine the graphs of the given equations to confirm the entries in the
   table.

## *Sine and Cosine Curves* (continued)

5. Graph these equations:   $y = \frac{1}{2}\sin(x)$,   $y = 2\cos(x)$

   Using $\pi$ when giving each period, complete the table below.

   | Equation | Period | Maximum | Minimum | Amplitude |
   |---|---|---|---|---|
   | $y = \frac{1}{2}\sin(x)$ | | | | |
   | $y = 2\cos(x)$ | | | | |

6. What effect does the positive constant $a$ seem to have on the graphs of $y = a\sin(x)$ and $y = a\cos(x)$?

   _____

7. Try to complete the table below before graphing the given equations.

   | Equation | Period | Maximum | Minimum | Amplitude |
   |---|---|---|---|---|
   | $y = 3\sin(x)$ | | | | |
   | $y = \frac{3}{2}\cos(x)$ | | | | |

   Now examine the graphs of the given equations to confirm the entries in the table.

8. Graph these equations:   $y = \sin(2x)$,   $y = \cos(\frac{1}{2}x)$

   Using $\pi$ when giving each period, complete the table below.

   | Equation | Period | Maximum | Minimum | Amplitude |
   |---|---|---|---|---|
   | $y = \sin(2x)$ | | | | |
   | $y = \cos(\frac{1}{2}x)$ | | | | |

9. What effect does the positive constant $b$ seem to have on the graphs of $y = \sin(bx)$ and $y = \cos(bx)$?

   _____

10. Try to complete the table below before graphing the given equations.

    | Equation | Period | Maximum | Minimum | Amplitude |
    |---|---|---|---|---|
    | $y = \cos(\frac{2}{3}x)$ | | | | |
    | $y = \sin(3x)$ | | | | |

    Now examine the graphs of the given equations to confirm the entries in the table.

11. Use the "Circular Function Quiz" program of Algebra Plotter Plus to see if you can correctly determine the equations of random sine and cosine curves drawn by the computer.

# Tangent, Cotangent, Secant, and Cosecant

Use the Function Plotter program of Algebra Plotter Plus. Change the
x-axis scale to multiples of π by selecting SCALE, moving the cursor to the
"Multiples of pi" option, and selecting "Yes."

1. Graph this equation: $y = \tan(x)$

   a. The graph has vertical *asymptotes*, which are lines that the graph
      approaches but does not cross. One such asymptote is the line $x = 0.5\pi$.
      Give the equations of all other asymptotes that appear on the screen.

      _____

   b. As the graph indicates, the tangent function is *periodic* (that is,
      repeating). You can determine the period by finding the distance
      between consecutive points where the graph crosses the x-axis. In terms
      of π, what is this period? _____

2. Graph this equation: $y = \cot(x)$

   a. The graph of the cotangent function has vertical asymptotes. Give the
      equations of all asymptotes that appear on the screen.

      _____

   b. In terms of π, what is the period of the cotangent function? _____

3. Graph these equations: $y = \sin(x)$

   $y = \csc(x)$

   The cosecant function is defined to be the reciprocal of the sine function.
   Use this fact as well as the graphs of the sine and cosecant functions to
   answer the questions.

   a. The range of $y = \sin(x)$ is $\{y: -1 \le y \le 1\}$. What is the range of
      $y = \csc(x)$?_____

   b. The period of the sine function is $2\pi$. What is the period of the cosecant
      function? _____

   c. Whenever the graph of $y = \sin(x)$ crosses the x-axis, what happens to the
      graph of $y = \csc(x)$?

      _____

4. Graph this equation: $y = \cos(x)$

   a. The secant function is defined to be the reciprocal of the cosine function.
      Use this fact as well as the results of Exercise 3 to sketch the graph of
      $y = \sec(x)$.

   b. Use the Function Plotter to graph $y = \sec(x)$ to confirm your sketch.

   c. Why are the graphs of $y = \cos(x)$ and $y = \sec(x)$ tangent to each other
      whenever x is a multiple of π?

      _____

      _____

*(continued)*

## *Tangent, Cotangent, Secant, and Cosecant* (continued)

5. Graph these equations:    $y = \tan(x)$

    $y = -1 + \tan(x)$

    $y = 2 + \tan(x)$

   Describe the effect of the constant $c$ on the graph of $y = c + \tan(x)$.

   _____

6. Using your answer to Exercise 5, sketch the graph of $y = -2 + \cot(x)$. Then graph $y = -2 + \cot(x)$ to confirm your sketch.

7. Graph these equations:    $y = \csc(x)$

    $y = 2\csc(x)$

    $y = \frac{1}{2}\csc(x)$

   Describe the effect of the positive constant $a$ on the graph of $y = a\csc(x)$.

   _____

8. Using your answer to Exercise 7, sketch the graph of $y = \frac{3}{2}\sec(x)$. Then graph $y = \frac{3}{2}\sec(x)$ to confirm your sketch.

9. Graph these equations:    $y = \cot(x)$

    $y = \cot(2x)$

    $y = \cot(\frac{1}{2}x)$

   Describe the effect of the positive constant $b$ on the graph of $y = \cot(bx)$.

   _____

10. Using your answer to Exercise 9, predict the period of $y = \tan(\frac{2}{3}x)$.  _____

    Graph $y = \tan(\frac{2}{3}x)$ to confirm your prediction.

# Histograms, Means, and Standard Deviation

| Algebra Plotter Plus | Book 2: Lesson 15-2 |

**Use the Statistics Spreadsheet program of Algebra Plotter Plus.**

1. At the right are the scores that a class of 16 students received on two tests.
   a. Select ENTER DATA. Using the arrow keys to move in the spreadsheet, replace the words "Column A" and "Column B" with the titles "Test 1" and "Test 2." Then enter the statistics at the right into the spreadsheet. Press <ESC> when you are done.
   b. Select GRAPH and highlight HISTOGRAM to plot a histogram for the scores from the first test. When the computer asks for the "Least value in first group," type 60. When the computer asks for the "Least value in second group," type 65. This will result in a histogram with bars for each interval of 5.
   c. Repeat part (b), this time using the scores from the second test,. (Return to the data and use the right-arrow key to highlight the second column. Use values of 70 and 75 to establish the interval of 5.)

|     | Test 1 | Test 2 |
|-----|--------|--------|
| 1.  | 60     | 84     |
| 2.  | 89     | 81     |
| 3.  | 65     | 77     |
| 4.  | 97     | 76     |
| 5.  | 98     | 80     |
| 6.  | 61     | 81     |
| 7.  | 62     | 82     |
| 8.  | 96     | 84     |
| 9.  | 71     | 86     |
| 10. | 66     | 88     |
| 11. | 94     | 85     |
| 12. | 95     | 72     |
| 13. | 93     | 74     |
| 14. | 63     | 77     |
| 15. | 88     | 81     |
| 16. | 98     | 88     |

2. Compare the two histograms from Exercise 1.
   a. In which graph are the scores clustered in the middle? _____
   b. In which graph are the scores out toward the edges? _____

3. Select CALCULATE to discover the range, mean, median, and standard deviation for each column.
   a. Which test has a larger range of scores? _____
   b. Which test has a mean equal to the median? _____
   c. How do the standard deviation statistics support your observations in Exercise 2? _____

4. Return to the spreadsheet and EDIT DATA to examine the following situations.
   a. Subtract 10 from each score of Test 1. How are the mean and standard deviation affected? _____
   b. Multiply each score of Test 2 by $\frac{1}{2}$. How are the mean and standard deviation affected? _____

# *Correlation and Line of Best Fit*  | Algebra Plotter Plus | | Book 2: Lesson 15-4 |

**Use the Statistics Spreadsheet program of Algebra Plotter Plus.**

1. The first table below gives six ordered pairs that satisfy the equation
   $y = 2x + 13$, while the second table below gives six ordered pairs that satisfy
   the equation $y = -2x + 13$.
   a. Use ENTER DATA to input the first set of ordered pairs into two
      columns of the spreadsheet. Then select GRAPH and SCATTER to see a
      plot of the pairs. Press the space bar to see the statistical data.
      What is the correlation coefficient for the first set? _____
   b. Repeat the procedure of part (a) with the second set of ordered pairs
      below.
      What is the correlation coefficient for the second set? _____

2. The third and fourth tables below are alterations of the first two so that,
   when graphed, the points are no longer collinear.
   a. Repeat the procedure of Exercise 1(a) with the third set of ordered pairs
      below.
      What is the correlation coefficient for the third set? _____
   b. Repeat the procedure of Exercise 1(a) with the fourth set of ordered pairs
      below.
      What is the correlation coefficient for the fourth set? _____

3. What conclusion can you draw from the results of Exercises 1 and 2?

   _____

   _____

4. The fifth table below shows the height and weight of a sample of small
   penguins. Enter the data into the spreadsheet, and select SCATTER to plot
   the pairs.
   a. What is the equation of the regression line (the line of best fit)?

   _____

   b. Explain why this line may not be a good model for the weight of very
      small penguins.

   _____

| x | y | x | y | x | y | x | y | height (cm) | weight (g) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 15 | 1 | 11 | 1 | 10 | 1 | 16 | 40 | 2011 |
| 2 | 17 | 2 | 9 | 2 | 22 | 2 | 4 | 38 | 2004 |
| 3 | 19 | 3 | 7 | 3 | 14 | 3 | 12 | 41 | 2107 |
| 4 | 21 | 4 | 5 | 4 | 26 | 4 | 0 | 32 | 1996 |
| 5 | 23 | 5 | 3 | 5 | 18 | 5 | 8 | 33 | 1915 |
| 6 | 25 | 6 | 1 | 6 | 30 | 6 | -4 | 26 | 1848 |

NAME _____    DATE _____

# *Sampling*     | Algebra Plotter Plus |  | Book 2: Lesson 15-9 Application |

**Use the Sampling Experiment program of Algebra Plotter Plus.**

1. Printed on each bag of Royal Wheat Flour is the claim that the bag contains
   2300 g of flour. An independent testing company weighs the contents of
   6 bags of flour and finds the mean weight to be only 2250 g, with a standard
   deviation of 100. To determine the likelihood of obtaining such a result,
   select the DEFINE option and enter the following numbers:

   |                              |                              |
   |------------------------------|------------------------------|
   | Population mean: 2300        | Value to test:  2250         |
   | Population standard deviation: 100 | Least sample mean shown:  2000 |
   | Sample size:  6              | Greatest sample mean shown:  2600 |

   Note that, since the standard deviation of the population is not known, you
   must assume that the sample standard deviation found by the testing
   company applies to the population as well.

2. Select SINGLE SAMPLE from the program menu. Have the program
   produce a random sample three times, and record the results below.

   |     | *Sample mean* | *Sample standard deviation* |
   |-----|---------------|------------------------------|
   | (1) | _____   | _____                  |
   | (2) | _____   | _____                  |
   | (3) | _____   | _____                  |

   Why do the sample means differ from one another and from the population
   mean? _____

3. You will learn more about the likelihood of a sample mean being 2250 g or
   lower by having the program produce a large number of samples. Select
   MANY SAMPLES, and let the program run until it reaches about 100
   samples. Record the following information:

   Percent less than 2250: _____

   Mean of sample means: _____        Standard deviation of sample means: _____

   Suppose that the percent of samples with mean less than 2250 is 10%. This
   means that even if the Royal Wheat Company's claim is true, about 10% of
   the samples of the size used by the testing company would have means less
   than 2250 g. A probability of 0.10 is not considered unlikely enough to
   prove that Royal Wheat is making a false claim.

4. To make a more definitive test of Royal Wheat's claim, the testing company
   weighs the contents of 25 bags of flour. The new sample mean is 2265 g,
   with a standard deviation of 95. Run the "Sampling Experiment" program
   with the new data for as long as it is practical to do so.
   a. What percent of the samples had a mean less than 2265? _____
   b. Are you more or less confident than you were in Exercise 3 that Royal
      Wheat is making a true claim when they say each bag contains an
      average of 2300 g? Explain why.

   _____

# Answers for Activities

## Algebra, Structure and Method, Book 1

### Graphs of Linear Equations in Standard Form
1. a. $3x - y = 1$   b. The $y$-intercept of each line is $-1$.
2. The first line falls; the second line rises. The $y$-intercepts of the two lines are opposites.   3. The $y$-intercept changes.
4. a. $2; -2; 1.5$   b. The $y$-intercept of $x + By = C$ is $\frac{C}{B}$.

### Slope-Intercept Form
1. up 2 units   2. down 3 units
3. a. slopes up to the right   b. slopes down to the right
4. 0   5. a. They are parallel.   b. The slopes are equal.
c. Parallel lines have equal slopes.   6. a. $2; -4; 1$   b. The $y$-intercept is $b$.   7. $-\frac{3}{2}, 0; 2, 1; 0, -3.5$

### Zeros of Functions
1. $\{-5\}; \{-1\}; \{4\}$   2. $\{-5, -4\};$
$\{-1, 1\}; \{4\}$   3. $\{-5, -4, -2\}; \{-1, 1, 2\}; \{4\}$   4. $\{-3, 0, 4\}$
5. $\{-2, -1, 1\}; \{-2, -1, 1\}; \{-2, -1, 1\};$ no   6. $\{-3, 2\}$
7. $\{-2, 2.5\}$   8. $\{-2.25, -0.5, 0, 1.5, 3\}$   9. See diagram on page 69. $\{-3, -0.5, \frac{2}{3}, 2\}$

### Inverse Variation
1. a. first and third quadrants
b. $y$-values get closer to zero   c. $y$-values get closer to zero
d. $y$-values get larger and larger   e. $y$-values get smaller and smaller   f. No; division by zero is undefined.
2. a. increase   b. decrease   3. a. $1, -1$   b. $6, -6$
c. See diagram on page 69. $(1, 6); (-1, -6)$

### Solving Systems of Linear Equations
1. $(2, -1)$   2. a. $(1.6, -1.2)$   b. $(1.66, -1.16)$
3. $(0.6, -0.4)$   4. $(-6.3, 0.4)$
5. a. The lines are parallel. There is no solution.   b. $4\frac{1}{2};$ All points on the line are solutions.

### Absolute Value Graphs
1. a. $(0, 1); (0, -2)$   b. $(0, C)$
2. a. $(1, 0); (-2, 0)$   b. $(-B, 0)$
3. $y - (-1) = (1)\text{Abs}(x - (3)), (3, -1); y - (2) = (1)\text{Abs}(x - (2)),$
$(2, 2); y - (-4) = (1)\text{Abs}(x - (-1)), (-1, -4)$   4. Answers may vary.   a. $y - 4 = |x + 2|$   b. $y + 3 = |x - 1|$

### Linear Programming
See diagram on page 69.
1. $(0, 0); (0, 8); (2, 7); (5, 4); (6, 0)$   2. a. $0, 8, 16$
b. $2, 7, 20$   c. $5, 4, 23$   d. $6, 0, 18$   e. $0, 0, 0$
f. $23, 5, 4$   3. a. $0, 0, 0$   b. $43, 2, 7$

## Algebra and Trigonometry, Structure and Method, Book 2

### Exploring Slopes
1. The lines get steeper.   2. a. $0.5;$
$(1, 0.5)$   b. $1; (1, 1)$   c. $2; (1, 2)$   d. For any line that passes through the origin, the point with $x$-coordinate 1 has a $y$-coordinate equal to the slope of the line.   3. a. Lines with positive slope rise from left to right; lines with negative slope fall from left to right.   b. $(0, 1)$   4. a. $-0.5; (0, 1); (1, 0.5)$
b. $-1; (0, 1); (1, 0)$   c. $-2; (0, 1); (1, -1)$

### Parallel and Perpendicular Lines
1. a. The lines are parallel.   b. The slopes are equal.   2. a. $y = -3x - 1$
b. The lines are parallel.   3. equal   4. a. The lines are parallel.   b. Answers may vary. For example: $A = 4, B = 3,$
$C = -9$.   5. Answers may vary. For example: $-\frac{A}{B} = -\frac{D}{E}$ or
$AE = BD$ or $AE - BD = 0$.   6. $y = \frac{2}{3}x$ and $y = -\frac{3}{2}x$
7. $y = 3x - 2$ and $y = -\frac{1}{3}x$   8. negative reciprocals
9. a. The lines are perpendicular.   b. Answers may vary.
For example, $A = 3, B = -4, C = 12$.   10. Answers may vary.
For example: $-\frac{A}{B} \cdot -\frac{D}{E} = -1$ or $BE + AD = 0$.

### Estimating Solutions of Linear Systems
1. $(3, 1)$
2. a. Answers may vary. For example: $(0.5, -1.5)$
b. Answers may vary. For example: $(0.4, -1.2)$
3. $(3.2, -0.1)$   4. $(-8.6, -18.1)$   5. By examining the differences between corresponding $y$-values in the table, you can find an $x$-value for which the difference is closest to 0. In this case, when $x = -8.625$, both $y$-values are $-18.125$. Therefore, the exact solution is $(-8.625, -18.125)$.

### Parabolas
1. $(-3, 0); (2, 0); (-1, 0)$   2. $(0, 2); (0, -1);$
$(0, 3)$   3. See diagram on page 69.
$y - (2) = (1)(x - (3))^2; (3, 2); y - (1) = (-3)(x - (-2))^2; (-2, 1);$
$y - (-3) = (2/3)(x - (-1))^2; (-1, -3)$   4. Answers may vary.
a. $y + 1 = (x - 2)^2$   b. $y + 1 = 2(x - 2)^2$   c. $y + 1 = 3(x - 2)^2$

### Approximating Roots of Polynomial Equations
1. a. $-2; -1$   b. $-1; 4$   c. $-2$   d. The $y$-value is closer to 0 for this $x$-value.   2. $-1.89$   3. a. 2   b. $-0.90; 1.10$
4. a. The root lies between the two consecutive $x$-values in the table for which the corresponding $y$-values are closest to 0. Use these $x$-values as the new $x$-axis scale boundaries for the next approximation.   b. $0.67$   5. $-0.33; 0.43$

## Identifying Central Conics

1. ellipse, $(1, -1)$; hyperbola, $(1, 1)$; circle, $(1, -2)$  2. circle, $(-1, 2)$; ellipse, $(2, -1)$; hyperbola, $(-2, -1)$  3. If $a = b$, then the graph is a circle. If $a \neq b$, then: graph is an ellipse if $a$ and $b$ have same signs; graph is a hyperbola if $a$ and $b$ have opposite signs.  4. hyperbola, $(-2, -2)$; circle, $(-1, 2)$; ellipse, $(2, -2)$

## Estimating Solutions of Quadratic Systems

1. Approximations may vary. For example: $(-0.5, 1.9)$; $(1.2, -1.5)$  2. $(-0.47, 1.94)$; $(1.27, -1.54)$  3. a. $(3.78, 1.31)$  b. $(-3.78, 1.31)$; $(-3.78, -1.31)$; $(3.78, -1.31)$  4. a. $-5y = 2$  b. The graph passes through the points of intersection of the given system.  5. See diagram on page 69.  a. $-7x + 3y = 5$  b. $(-0.21, 1.19)$; $(-2.35, -3.81)$

## Exponential Growth

1. a. $y = 2^x$  b. $y = 4^x$

2. a. $\left(\frac{3}{2}\right)^x$  b. $\left(\frac{2}{3}\right)^x$  3. a. increasing  b. decreasing

4. a. When $b > 1$, $b^x$ is increasing.  b. When $0 < b < 1$, $b^x$ is decreasing.  c. When $b = 1$, $b^x$ remains constant.  5. a. 80; 160; 320  b. $y = 5(2)^x$  c. 5120; 20480  d. $0 \leq y \leq 21000$  e. See diagram on page 69. Estimates may vary. For example: $\approx 28$  f. 3  6. a. $\approx 40$; $\approx 41$; $\approx 43$  b. yes  d. $\approx 54$  e. $\approx 7.5$  f. $\approx 18$  g. $\approx 36$; yes  7. $\approx 12$ years  8. $\approx 18$ years  9. a. $\approx 15$ years  10. a. $\approx 23$  b. $\approx 22$  c. $y = 24(0.97)^x$  d. yes  11. a. $y = 40(0.96)^x$  c. $\approx 17$  d. $\approx 8$

## Functions and Their Inverses

1. The graph of $f(x)$ appears to be the reflection of $g(x)$ in the line $y = x$.  2. They are reflections of each other in the line $y = x$.  3. See diagram on page 70.  4. $y = \pm\sqrt{x + 4}$ is not a function, so Function Plotter cannot plot it except by plotting the two functions $y = \sqrt{x + 4}$ and $y = -\sqrt{x + 4}$.

## Sine and Cosine Curves

1. a. $2\pi$; $2\pi$  b. $1, -1$; $1, -1$  c. $1$; $1$  d. $\frac{\pi}{2}$  2. $2\pi, 2, 0, 1$; $2\pi, -1, -3, 1$  3. shifts the curve $c$ units vertically  4. $2\pi, -2, -4, 1$; $2\pi, 5, 3, 1$  5. $2\pi, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$; $2\pi, 2, -2, 2$  6. changes maximum, minimum, and amplitude by a factor of $a$  7. $2\pi, 3, -3, 3$; $2\pi, \frac{3}{2}, -\frac{3}{2}, \frac{3}{2}$  8. $\pi, 1, -1, 1$; $4\pi, 1, -1, 1$  9. changes period to $\frac{2\pi}{b}$  10. See diagram on page 70.  $3\pi, 1, -1, 1$; $\frac{2}{3}\pi, 1, -1, 1$

## Tangent, Cotangent, Secant, and Cosecant

1. a. $x = -1.5\pi$; $x = -0.5\pi$; $x = 0.5\pi$; $x = 1.5\pi$  b. $\pi$  2. a. $x = -2\pi$; $x = -1\pi$; $x = 0\pi$; $x = 1\pi$; $x = 2\pi$  b. $\pi$  3. a. $\{y: y \leq -1 \text{ or } y \geq 1\}$  b. $2\pi$  c. It approaches a vertical asymptote.  4. c. Whenever $x$ is a multiple of $\pi$, $\cos(x) = 1$ or $\cos(x) = -1$. If $\cos(x) = 1$, then $\sec(x) = \frac{1}{1} = 1$. If $\cos(x) = -1$, then $\sec(x) = \frac{1}{-1} = -1$. Thus, whenever $x$ is a multiple of $\pi$, $\cos(x) = \sec(x)$.  5. shifts the graph $c$ units vertically  7. Stretches or shrinks the graph vertically; for example, if $|a| < 1$, then the graph looks more "squared off" near the $x$-axis.  9. changes period from $\pi$ to $\frac{\pi}{b}$  10. period $= 1.5\pi$

## Histograms, Means, and Standard Deviation

2. a. Test 2  b. Test 1  3. a. Test 2  b. Test 2  c. The standard deviation is greater in Test 1 than in Test 2; this supports the observation that the scores are less centrally clustered in Test 1 than in Test 2.  4. a. The mean is decreased by 10; the standard deviation is unchanged.  b. The mean and the standard deviation are multiplied by $\frac{1}{2}$.
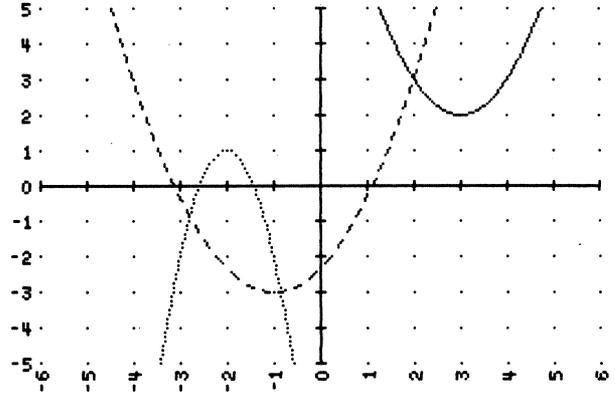
## Correlation and Line of Best Fit

1. a. 1  b. -1  2. a. 0.71428571  b. -0.7142857  3. Collinear points have a correlation coefficient of 1 (for lines with positive slope) or -1 (for lines with negative slope). As the graphs become less collinear, the correlation coefficient approaches 0.  4. See diagram on page 70.  a. $y = (13.77439)x + (1498.063)$  b. A penguin of height 0 cm would weigh 1498 g according to the linear model.

## Sampling

Results for these exercises are generated randomly by the computer. Examples are given.  2. (1) 2283.74, 92.051707; (2) 2302.9373, 97.654152; (3) 2310.8776, 97.774145. Because each sample does not contain the entire population, its mean may differ from the population mean. Also, because different samples may contain different elements of the population, the sample means may differ from one another.  3. 10.78%; 2300.3189; 40.30533  4. See diagram on page 70.  a. 3.46%  b. You might be less confident that Royal Wheat is making a true claim, because only about 3.46% of such 25-bag samples would have a mean of 2265 or less. It is unlikely that the testing company would have at random selected one of these samples. A possible hypothesis is that Royal Wheat is putting less than 2300 g of wheat in each bag on average.
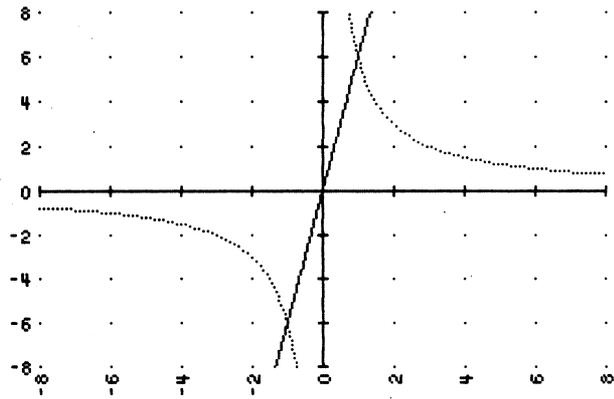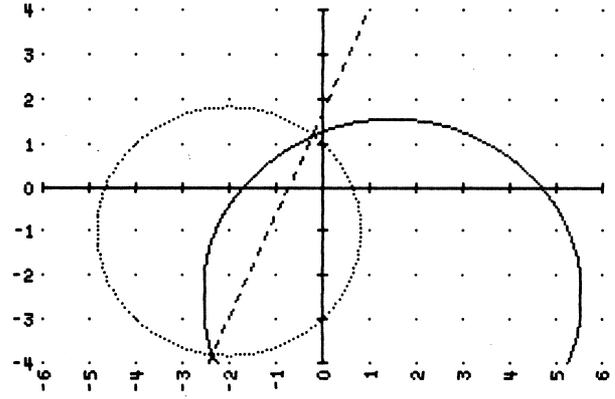
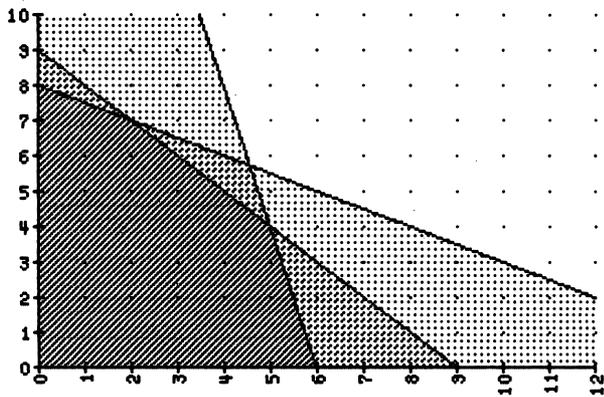**Zeros of Functions** Exercise 9
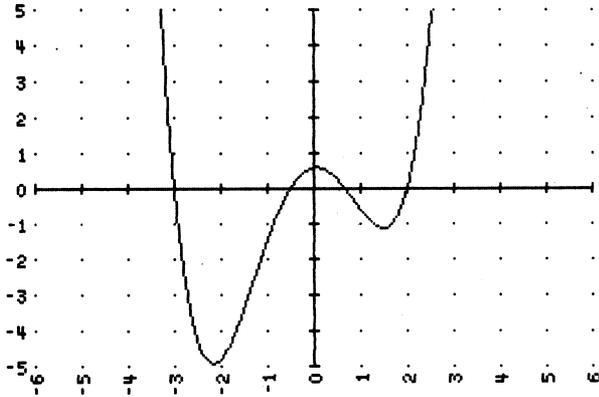


**Parabolas** Exercise 3



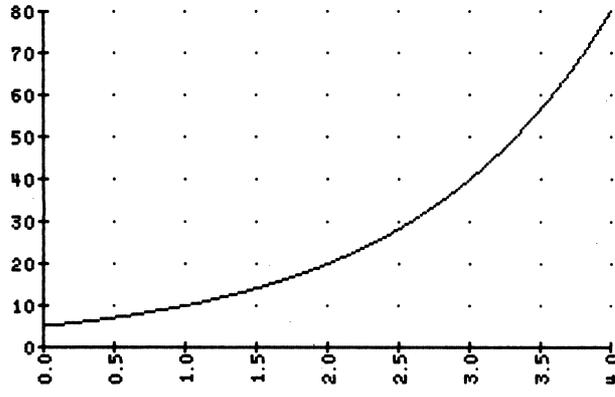**Inverse Variation** Exercise 3(c)



**Estimating Solutions of Quadratic Systems** Ex. 5
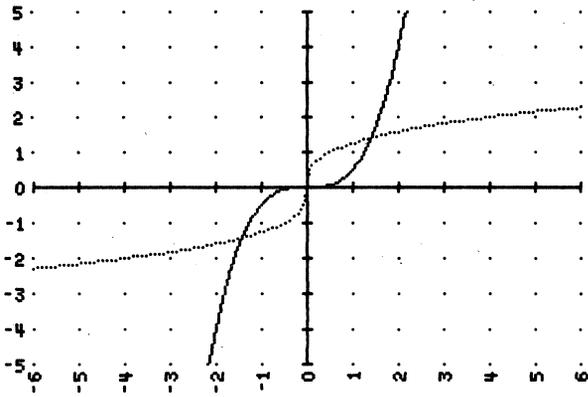


**Linear Programming** Exercises 1-3



**Exponential Growth** Exercise 5(e)
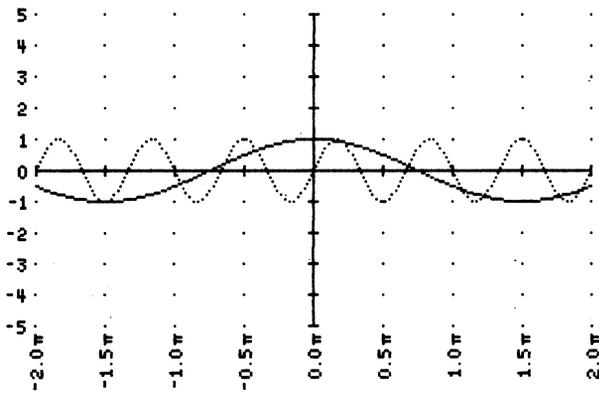
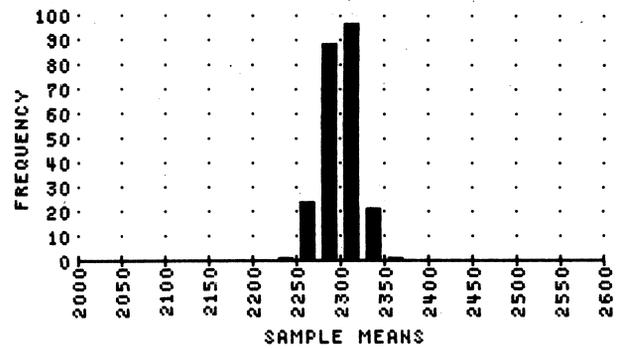## Functions and Their Inverses Exercise 3



## Sine and Cosine Curves Exercise 10



## Correlation and Line of Best Fit Exercise 4
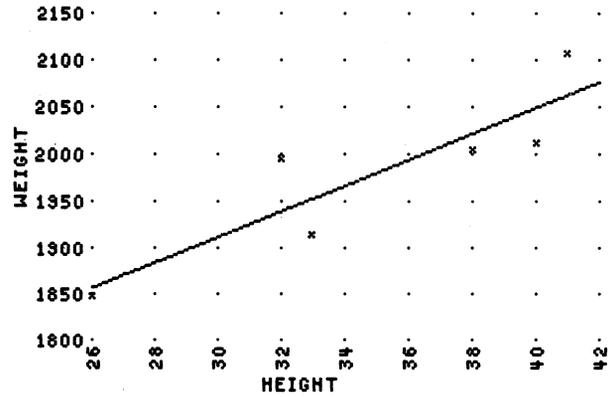


Regression line: y = (13.77439)x + (1498.063)

## Sampling Exercise 4



Percent less than 2265: 3.4632035%

## Correlation and Line of Best Fit Exercise 4

| Item Label | height | weight |
|------------|--------|--------|
| 1. Penguin A | 40 | 2011 |
| 2. Penguin B | 38 | 2004 |
| 3. Penguin C | 41 | 2107 |
| 4. Penguin D | 32 | 1996 |
| 5. Penguin E | 33 | 1915 |
| 6. Penguin F | 26 | 1848 |
| Number | 6 | 6 |
| Least | 26 | 1848 |
| Greatest | 41 | 2107 |
| Median | 35.5 | 2000 |
| Mean | 35 | 1980.1667 |
| Std Dev | 5.228129 | 81.210463 |

Brown
Dolciani
Sorgenfrey
Cole
Kane